Equilibrium Technology Diffusion, Trade, and Growth

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ABSTRACT

We study how opening to trade affects economic growth in a model where heterogeneous firms can choose to adopt a new technology already in use by other firms. We characterize the growth rate using summary statistics of the profit distribution—the ratio of profits between the average and marginal adopting firm. Opening to trade increases the spread in profits through increased export opportunities and foreign competition, induces more rapid technology adoption, and generates faster growth. Quantitatively, opening to trade yields large increases in growth, but welfare effects are muted due to loss of variety and reallocation of labor away from production.

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1. Introduction

A large body of evidence documents trade-induced productivity effects at the firm level (see, e.g., Pavcnik (2002) and Holmes and Schmitz (2010)). Why does opening to trade lead to productivity gains at the firm level? What are the consequences of these within-firm productivity gains for aggregate economic growth and welfare?

This paper contributes new answers to these questions. We develop a model where heterogeneous firms choose either to produce with their existing technology or adopt a better technology already in use by other firms. These choices determine the productivity distribution from which firms can acquire new technologies and, hence, the equilibrium rate of technological diffusion and economic growth. We provide a closed form characterization of the economy showing how the reallocation effects of a trade liberalization (i.e., low productivity firms contract or exit, high-productivity exporting firms expand) change firms' incentives to adopt a better technology and lead to faster within-firm productivity gains. Because these choices lead to more adoption and technology diffusion the aggregate consequence is faster economic growth.

The starting point of our analysis is a standard heterogeneous firm model in differentiated product markets as in Melitz (2003). Firms are monopolistic competitors who differ in their productivity/technology and have the opportunity to export after paying a fixed cost. There is free entry from a large mass of potentially active firms and firms exit at an exogenous rate. Our model of technology adoption and diffusion builds on Perla and Tonetti (2014), where firms choose to either upgrade their technology or continue to produce with their existing technology in order to maximize expected discounted profits for the infinite horizon. If a firm decides to upgrade its technology, it pays a fixed cost in return for a random productivity draw from the equilibrium distribution of firms within the domestic economy. We interpret this process as technology diffusion, since firms upgrade by adopting technologies already in use by other firms. Economic growth is a result, as firms are continually able to upgrade their technology by imitating other, better firms in the economy. Thus, this is a model of growth driven by endogenous technology diffusion.

We study how opening to trade affects firms' technology choices and the aggregate consequences for growth and welfare. To do so we characterize the profit and value functions of a firm, the evolution of the productivity distribution, and the growth rate of the economy on the balanced growth path equilibrium. We then study several issues: how changes in iceberg trade costs affect growth rates, the welfare gains from trade, and the model implied dynamics of a firm (during both normal times and trade liberalizations) in comparison to the large body of evidence on firm/establishment dynamics.

We provide a closed form characterization of the growth rate as a simple, increasing function of summary statistics of the profit distribution—the ratio of profits between the average and marginal adopting firm. A firm's incentive to adopt depends on two competing forces: the expected benefit of a new productivity draw and the opportunity cost of taking that draw. The expected benefit relates to the profits that the average new technology would yield. The opportunity cost of adopting a new technology is the forgone profits from producing with the current technology. Thus, the aggregate growth rate of the economy encodes the trade-off that firms face in a simple and intuitive manner.

Reductions in iceberg trade costs increase the rate of technology adoption and economic growth because they widen the ratio of profits between the average and marginal adopting firm. As trade costs decline, low productivity firms contract as competition from foreign firms reduce their profits; high productivity firms expand and export, increasing their profits. For low productivity firms, this process reduces both the opportunity cost and weakly increases the benefit of a new technology. This leads to more frequent technology adoption at the firm level. Because the frequency at which firms upgrade their technology is intimately tied to aggregate growth, the growth rate is higher in more open economies.

The underlying mechanism in our model is distinct from the standard "market size" effect, i.e., opening to trade increases the size of the market and, hence, raises the value of adoption. We show this by studying a special case of our model with no fixed cost in which all firms export. In this model, growth is the same function of the spread in profits between the average and marginal adopting firm. The difference is that trade has no effect on growth. In this model, opening to trade benefits all firms by increasing firms' profits and values by the same proportional amount. Consistent with the well understood benefits of a larger market, opening to trade increases the expected value of adopting a new technology. However, a larger market also raises the forgone profits of adoption by the same exact amount. Thus, opening to trade does not affect the *relative* benefit of adoption and, hence, there is no change in growth.

We provide a closed form characterization of the change in welfare from these growth effects. The change in welfare is a weighted sum of the increase in economic growth and the change in the initial level of consumption. The change in consumption is a sum of three components: a static gain from trade as in Arkolakis, Costinot, and Rodriguez-Clare (2012), a change in the mass of varieties consumed, and a change in the amount of labor allocated to the production of goods. We prove that opening to trade reduces the initial level of consumption and dampens the gains from faster economic growth. Similar to the quantitative results in Atkeson and Burstein (2010), we prove that the static gains from trade are always offset by a loss in varieties produced and consumed and a reallocation of labor away from goods production towards entry and adoption activity. Varieties decrease because the expected benefits of entering rise by less than the expected cost of entry—an increase in wages increases the cost of entry for all entering firms, but only some of the more productive firms benefit from lower trade costs. This results in a decrease in the level of initial consumption.

How faster growth competes with the loss in consumption is a quantitative question. To answer this question, we calibrate the parameters of the model to match aggregate trade volumes, growth, and properties of the firm size distribution. A move from autarky to the observed volume of trade leads to 0.60 percentage points higher economic growth and welfare is 13 percent higher. To put this number in perspective, it is nearly equivalent to the gains a static Arkolakis, Costinot, and Rodriguez-Clare (2012) measurement would deliver. The reason is that the dynamic gains from trade do not come for free. These trade-induced within-firm productivity improvements and their aggregate growth effects come with costs—and these costs take the form of losses in variety and reallocation of resources away from goods production.

1.1. Related Literature

We contribute to the theoretical literature on trade and growth. The standard mechanisms creating a relationship between trade and growth typically take two forms. First, openness leads to the cross-country diffusion of new and better ideas. Second, opening to trade increases the size of the market and, hence, raises the value of new idea creation/innovation. Depending on the details of the model, these mechanisms have been shown to increase economic growth as a country opens up to trade (see, for example, the pioneering works of Rivera-Batiz and Romer (1991) and Grossman and Helpman (1993) and their extensions to heterogeneous firm environments in Baldwin and Robert-Nicoud (2008)).

Our model differs from these traditional mechanisms. First, to focus on our distinct mechanism, we deliberately shut down the cross-country diffusion of new and better ideas. In our model, firms only acquire ideas already present inside their country. Thus, our model delivers growth without any increase in the amount or quality of ideas as a country opens to trade. This distinction is also salient relative to recent work such as Alvarez, Buera, and Lucas (2014) and Buera and Oberfield (2015). Second, as mentioned above, when only market size effects are present, opening to trade has no effect on economic growth. The relationship between growth and trade is not because a larger market increases the value of adoption; it's because of a relative change in the value of adoption that arises because of a trade liberalization's differential effects on firms.

More broadly, we relate to the literature on the relationship between competition and productivity. Arrow's (1962) "replacement effect" is a theoretical explanation for the positive effects of competition on adoption. Arrow's (1962) idea is that because a monopolist restricts output relative to a competitive industry, the monopolist is less willing to pay the fixed cost to improve efficiency since there are fewer units to spread the cost over. This explanation is problematic within the context of a trade liberalization. Arrow's (1962) logic implies that if trade reduces the output of a firm—as is typical for import-competing firms—then adoption should decrease (see Demsetz, 1969), which is not consistent with the empirical evidence discussed below.

Our model avoids the market size critique of Arrow (1962). The reason is that competition reduces the opportunity cost of adoption. As our theoretical results make transparent, the adoption decision and aggregate growth rate depend on the comparison between the potential benefits of adoption verses the forgone profits of operating with the old technology. On its own, the erosion of profits from import competition incentivizes firms to adopt more frequently. In this sense, our model shares similarities with Holmes, Levine, and Schmitz Jr (2012) who show how competition reduces the cost of a switch-over disruption from a new technology and leads to more technology adoption.

Closely related to our work is Bloom, Romer, Terry, and Van Reenen (2014) which focuses explicitly on import competition, within-firm productivity improvements, and aggregate growth. Motivated by the evidence in Bloom, Draca, and Reenen (2015), they show import competition forces firms to innovate more than otherwise. While similar in spirit, the underlying mechanisms are different. Central to their results is the costly adjustment of factors of production within the firm. As firms face import competition, the resources within the firm that are costly to shed are redirected toward innovative activities. Furthermore, their mechanism only amplifies and is not distinct from the traditional market size effects on innovation.

Our normative results share similarities to Atkeson and Burstein (2010). In a very different model of innovation, Atkeson and Burstein (2010) show how the welfare losses from the entry margin or "product innovation" offset the gains from within-firm innovation or "process innovation" in their language. In our model, the analog of this result are the drags on welfare from a loss in variety. Our positive results, however, are different. In Atkeson and Burstein (2010), it is the large, exporting firms that innovate more and small import-competing firms that reduce their innovation. In our model, it is the small import-competing firms that speed up their adoption of better technologies. As our model focuses on adoption and does not feature innovation, these represent different mechanisms contributing to welfare.

We also contribute to the literature on idea flow models of economic growth in several ways. The most important is the introduction and study of competition effects which are new, additional, forces not present in the endowment economies of Lucas and Moll (2014) and Perla and Tonetti (2014).[1] As our closed form characterization of the growth rate shows, without any relative change in firms' profits, opening to trade has no effect on economic growth. Thus the competition effects that we introduce, which act through a reallocation of profits, are key to delivering interesting relationships between trade, firms' technology choices, and economic

[1]Even in the closed economy version of this model, we extend Perla and Tonetti (2014) in important directions: general equilibrium with labor and goods markets in continuous time with firm entry and exit. Also, our characterization of growth as a function of summary statistics of the profit distribution is a core contribution and generalizes Perla and Tonetti (2014). Finally, our Appendix solves the model with firm-specific geometric Brownian motion shocks to productivity which may be of use for empirical applications using firm-level data.

growth.

Sampson (2015) studies the effects of trade on growth when there is a dynamic complementarity between the ideas of entrants and those of the incumbents: trade induces exit of the worst performing firms and this implies that entrants are able to receive better ideas; because entrants are now better, this induces more selection and so on, leading to faster economic growth. While our model has entry and exit, it is incumbents that adopt new technologies. This distinction is empirically relevant as Sampson's (2015) model—following Luttmer (2007)—is one in which most of aggregate productivity growth (and its response to trade) is from the entry margin. In contrast, our model implies that most of aggregate productivity growth comes from within-firm improvements by incumbents, as is indicated by Garcia-Macia, Hsieh, and Klenow (2015).

1.2. Motivating Evidence: Trade-Induced Productivity Gains

Motivating our work is the empirical evidence that import competition gives rise to within-firm productivity improvements.[2]

Pavcnik (2002) was an important empirical study of the establishment level productivity effects from a trade liberalization using frontier measurement techniques. Pavcnik (2002) studied Chile's trade liberalization in the late 1970s and she found large, within-plant productivity improvements for import-competing firms that are attributable to trade. There was no evidence that exporters had any productivity improvements attributable to trade and no evidence of trade induced productivity gains from exit. To be clear, Pavcnik (2002) observed productivity improvements from exit, but there were no *differential* gains from exit across sectors of different trade orientation (i.e., import competing vs. non-traded, etc.). In contrast, import-competing firms had differentially larger within-plant productivity improvements.

Many subsequent studies for different countries and/or data sets have found similar results. In Brazil, Muendler (2004) found import competition led to within firm productivity gains. Several studies of India's trade liberalization find related results. Topalova and Khandelwal (2011) found large within-firm productivity gains associated with declines in output tariffs which proxy for increases in import competition. Also in India, Sivadasan (2009) finds increases in industry TFP from tariff reductions, with 55 percent of these gains associated with within-firm productivity gains.

Bloom, Draca, and Reenen (2015) find within-firm productivity gains in Europe from Chinese import competition. Most importantly, they associate these gains with explicit measures of technical change, e.g., information technology, management practices, and other measures of

[2] There are aspects of firm-level adjustments to trade liberalizations that we have little to say about. In particular, the evidence on the productivity enhancing role of becoming an exporter (see, e.g., Bustos (2011) or Marin and Voigtländer (2013)).

innovation. Their evidence suggests that firms undertook activities to change the technology with which they operate in response to import competition.

Despite the large body of empirical work, theory has lagged.[3] Two common explanations for these within-firm productivity gains fall under the category of imperfect measurement. The first explanation is that these gains may reflect changes in the mix of intermediate inputs. For the cases of Indonesia (studied in Amiti and Konings, 2007) and India (Goldberg, Khandelwal, Pavcnik, and Topalova, 2010), there is strong evidence for this mechanism. A second explanation is that they reflect changes in product mix (see, e.g., Bernard, Redding, and Schott, 2011). While these are likely contributing forces, there is evidence they are not the whole story. For example, Bloom, Draca, and Reenen (2015) find little evidence that they are the source of the gains in their study.

Non-measurement explanations fall under the guise of "X-efficiency" gains (Leibenstein, 1966). X-efficiency gains can be difficult to understand since it is natural to ask the question: If it was possible for a firm to improve its efficiency after a change in competition, why did it not do it in the first place? One mechanism for X-efficiency gains is that competition relaxes the agency problems within the firm (see, e.g., Schmidt, 1997; Raith, 2003). In our model, increased import competition increases the profitability of technological improvement by lowering the opportunity cost of adoption relative to the returns of adoption. This competition driven increase in the pace of technology adoption leads to within-firm productivity gains that generate faster aggregate economic growth.

2. Model

2.1. Countries, Time, Consumers

There are N symmetric countries. Time is continuous and evolves for the infinite horizon. Utility of the representative consumer in country i is

$$U_i(t) = \int_t^{\infty} e^{-\rho(\tau - t)} \log(C_i(\tau)) d\tau. \tag{1}$$

The utility function $U_i(t)$ is the present discounted value of the instantaneous utility of consuming the final good. The discount rate is $\rho > 0$ and instantaneous utility is logarithmic.[4] The final consumption good is an aggregate bundle of varieties, aggregated with a constant elasticity of

[3] The standard heterogeneous firm framework of Melitz (2003) does not provide an explanation for these effects. Melitz (2003) deals exclusively with the reallocation of activity across firms; there is no mechanism to generate within-firm productivity growth in response to a trade liberalization.

[4] The model easily generalizes to CRRA power utility, as shown in the Appendix, but analytical characterizations are less sharp.

substitution (CES) function by a competitive final goods producer.

Consumers supply labor to firms for the production of varieties, the fixed cost of exporting, and possibly the fixed costs for technology adoption and entry. Labor is supplied inelastically and the total units of labor in a country are $\bar{L}_i$. Consumers also own the firms operating within their country and, thus, their income is the sum of profits and total payments to labor.

We abstract from borrowing or lending decisions, so consumers face the following budget constraint

$$P_i(t)C_i(t) = W_i(t)\bar{L}_i + P_i(t)\bar{\Pi}_i(t), \tag{2}$$

where $W_i(t)$ is the nominal wage rate in country i, $P_i(t)$ is the standard CES price index of the aggregate consumption good, and $\bar{\Pi}_i(t)$ is real aggregate profits (net of investment costs) in consumption units. These relationships are elaborated in detail below.

2.2. Firms

In each country there is a final good producer that supplies the aggregate consumption good competitively. The final good is produced by aggregating an endogenous mass of intermediate varieties produced by monopolistically competitive firms, both domestically and abroad. Variety producing firms are heterogeneous over productivity, Z, with cumulative distribution function $\Phi_i(Z,t)$ describing how productivity varies across firms, within a country. Each firm alone can supply variety v. As is standard, a final good producer aggregates these individual varieties using a constant elasticity of substitution production function.

2.2.1. Final Good Producer.

Dropping the time index for expositional clarity, the final good producer chooses the quantity to purchase of each variety:

$$\max_{Q_{ij}(v)} \left[\sum_{j=1}^{N} \int_{\Omega_{ij}} Q_{ij}(v)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \tag{3}$$

$$\text{s.t.} \sum_{j=1}^{N} \int_{\Omega_{ij}} p_{ij}(v) Q_{ij}(v) = Y_i.$$

Y_i is defined to be nominal aggregate expenditures on consumption goods. The parameter $\sigma > 1$ is the elasticity of substitution across varieties. The measure Ω_{ij} defines the endogenous set of varieties consumed in country i produced in country j. Furthermore, the total mass of varieties produced in country i, $\Omega_i(t)$, is also determined in equilibrium, as domestic firms can enter after paying a fixed cost and exit if hit with an exogenous death shock that arrives at rate $\delta \geq 0$.

We will drop the notation carrying around the variety identifier, as it is sufficient to identify each firm with its location and productivity level, Z. Additionally, to focus on the interactions between technology adoption, trade, and growth, we assume that all countries are symmetric in that they have identical parameters, although each intermediate producer in each country produces a unique good. Because all countries are symmetric, we focus on the results for a typical country and abstract from notation indicating the country's location.

This final good producer problem yields the familiar variety demand and price index equations:

$$Q(Z) = \left(\tfrac{p(Z)}{P}\right)^{-\sigma} \tfrac{Y}{P}, \tag{4}$$

$$P^{1-\sigma} = \Omega \left(\int_M^\infty p_d(Z)^{1-\sigma} \mathrm{d}\Phi(Z) + (N-1) \int_{\hat{Z}}^\infty p_x(Z)^{1-\sigma} \mathrm{d}\Phi(Z) \right). \tag{5}$$

where p_d and p_x are the prices of domestic and imported varieties, M is the minimum of support of the distribution, and $\hat{Z}$ is an export threshold—all determined endogenously.

2.2.2. Individual Variety Producers.

Variety producing firms hire labor, ℓ, to produce quantity Q with a linear production technology: $Q = Z\ell$. Firms can sell freely in their domestic market and also have the ability to export at some cost, controlled by parameter κ. To export, a firm must pay a fixed flow labor cost, $\kappa \bar{L} W/P$, per foreign export market. This fixed cost is paid in market real wages and is proportional to the number of consumers accessed.[5] Exporting firms also face iceberg trade costs, $d \geq 1$, to ship goods abroad.

Furthermore, at each instant, any firm can pay a real fixed cost $X(t)$ to adopt a new technology. $X(t)$ represents the cost of hiring labor to upgrade to higher-efficiency production technologies. Similar to the fixed cost of export, the fixed cost of adoption takes the form $X(t) = \zeta \bar{L} W/P$, controlled by parameter ζ. If a firm decides to pay this cost, it receives a random draw from the distribution of producers within its own country, as in Perla and Tonetti (2014).[6]

[5]Export costs that are proportional to the number of consumers is consistent with the customer access interpretation featured in Arkolakis (2010). As discussed in Section 4, this influences the population scale effect properties of the model, but has no other impact.

[6]Since countries are symmetric, cross-country technology diffusion modeled as a mixture of draws across countries is identical.

Given this environment, firms must make choices regarding how much to produce, how to price their product, whether to export, and whether to change their technology. These choices can be separated into problems that are static and dynamic. Below we first describe the more standard static problem of a firm and then describe the dynamic problem of the firm to derive the optimal technology adoption policy.

Firms' Static Problem. Given a firm's location, productivity level, and product demand, the firm's static decision is to chose the amount of labor to hire, the prices to set, and whether to export for each destination to maximize profits each instant. The firm's problem when operating within the domestic market is to choose a price and quantity of labor to maximize profits. Using the standard demand function for individual varieties (equation 4), the optimal domestic real profit function is

$$\Pi_d(Z) = \frac{1}{\sigma}\left(\frac{\bar{\sigma}W}{ZP}\right)^{1-\sigma}\frac{Y}{P}, \quad \text{where} \quad \bar{\sigma} := \frac{\sigma}{\sigma-1}. \tag{6}$$

$\bar{\sigma}$ is the standard markup over marginal cost.

The decision to (possibly) operate in an export market is similar, but differs in that the firm faces variable iceberg trade costs and a fixed cost to sell in the foreign market. Optimal per-market real export profits are

$$\Pi_x(Z) = \max\left\{0, \ \frac{1}{\sigma}\left(\frac{\bar{\sigma}dW}{ZP}\right)^{1-\sigma}\frac{Y}{P} - \frac{\kappa\bar{L}W}{P}\right\}. \tag{7}$$

where d is an iceberg trade costs and $\kappa\bar{L}W/P$ is the fixed cost to sell in the foreign market. Given profits described in equation (7), only firms earning positive profits from exporting—those above a productivity threshold $\hat{Z}$—actually enter a foreign market. Total real firm profits equal the sum of domestic profits plus the sum of exporting profits across export markets,

$$\Pi(Z,t) := \Pi_d(Z,t) + (N-1)\Pi_x(Z,t). \tag{8}$$

Firms' Dynamic Problem. Given the static profit functions and a perceived law of motion for the productivity distribution and adoption cost, each firm has the choice of when to acquire a new technology, Z. Define $g(t)$ as the growth rate of total expenditures.[7] Since firms are owned by consumers, they choose technology adoption policies to maximize the present discounted expected value of real profits, discounting with interest rate $r(t) = \rho + g(t) + \delta$.

[7]Since in equilibrium many growth rates will be equal (e.g., the growth rate of total expenditures, consumption, and the minimum of the productivity distribution), we will abuse notation for the sake of exposition and overload the definition of a single growth rate: $g(t)$.

The recursive formulation of the firm's problem is as follows. Each instant, a firm with productivity Z chooses between continuing with its existing technology and earning flow profits of $\Pi(Z,t)$ or stopping and adopting a new technology at cost $X(t)$. In a growing economy, adoption opportunities will improve and the firm's profits will erode, decreasing the benefits of continuing to operate its existing technology until the firm enters the stopping region and it chooses to adopt a new technology.

Define the value of the firm in the continuation region as $V(Z,t)$, $M(t)$ as the time dependent productivity boundary between continuation and adoption, and $V_s(t)$ as the expected value of adoption net of costs. $M(t)$ is a reservation productivity function, such that all firms with productivity less than or equal to $M(t)$ choose to adopt and all other firms produce with their existing technology. If a firm chooses to adopt a new technology it pays a cost and immediately receives a new productivity. This new productivity is a random draw from the cross-sectional productivity distribution of firms.[8] This distribution will be a function of the optimal policy of all firms, i.e., the firm choice of when to draw a new productivity. Recursively, the optimal policy of firms will depend on the expected evolution of this distribution. With rational expectations, the expected net value of adoption in equilibrium is

$$V_s(t) = \int_{M(t)}^{\infty} V(Z,t)\mathrm{d}\Phi(Z,t) - X(t). \tag{9}$$

There are several interpretations of this technology adoption choice. The literal interpretation is that people are randomly matching and learning from each other. Empirically, this technology choice can be thought of as tangible or intangible investments that manifest themselves as improvements in productivity like improved production practices, work practices, supply-chain and inventory management, etc. that are already in use by other firms (see, for example, the discussions in Holmes and Schmitz (2010) and Syverson (2011)).

Using the connection between optimal stopping and free boundary problems, a set of partial differential equations (PDEs) and boundary conditions characterize the firm's value.[9] The PDEs

[8] In discrete time, Perla and Tonetti (2014) presents both draws conditional on only meeting adopters and unconditionally from the whole distribution. Qualitatively, these two environments are very similar and in the limit to continuous time they become identical. See Benhabib, Perla, and Tonetti (2015) for a discussion and a proof that the unconditional and conditional draw models generate identical equilibrium laws of motion for the productivity distribution.

[9] Standard references and conditions for the equivalence between optimal stopping of stochastic processes and free boundary problems are Dixit and Pindyck (1994) and Peskir and Shiryaev (2006). The deterministic stopping problem presented in the main body of this paper is discussed on pages 110-115 of Stokey (2009). A more general problem with exogenous productivity shocks that follow geometric Brownian motion (GBM) is derived in Appendix A.3.

and boundary values determining a firm's value are

$$r(t)V(Z,t) = \Pi(Z,t) + \frac{\partial V(Z,t)}{\partial t}, \tag{10}$$

$$V(M(t),t) = \int_{M(t)}^{\infty} V(Z,t)\mathrm{d}\Phi(Z,t) - X(t), \tag{11}$$

$$\frac{\partial V(M(t),t)}{\partial Z} = 0. \tag{12}$$

Equation (10) describes how the firm's value function evolves in the continuation region. It says that the flow value of the firm equals the flow value of profits plus the change in the value of the firm over time. Equation (11) is the value matching condition. It says that at the reservation productivity level, $M(t)$, the firm should be indifferent between continuing to operate with its existing technology and adopting a new technology. Equation (12) is the smooth-pasting condition. The smooth pasting condition can be interpreted as an intertemporal no-arbitrage condition that ensures the recursive system of equations is equivalent to the fundamental optimal stopping problem.

A couple of comments are in order regarding the economics of this problem.

There are two forces that drive the adoption decision. First, over time the productivity distribution will improve. This eventually gives firms an incentive to adopt a new technology as the benefit of adoption grows over time. This economic force is the same as in Lucas and Moll (2014) and Perla and Tonetti (2014).

Second, competition and general equilibrium effects are new, additional, forces—not present in Lucas and Moll (2014) and Perla and Tonetti (2014)—which drive the adoption decision. The dependence of the firm's value function (equation 10) on profits (which are time dependent) illustrates this feature. As an economy grows, an individual firm's profits will erode. The reason is because as other firms become better through adoption, they demand relatively more labor, and this raises wages which reduces the profits of non-adopting firms. This erosion of profits reduces the opportunity cost of continuing to operate and incentivizes adoption. Our paper is about this second force—how equilibrium changes in competition and profits via trade influence adoption and growth.

Finally, there is an externality in this environment. Firms are infinitesimal and do not internalize the effect their technology adoption decisions have on the evolution of the productivity distribution and, in turn, the distribution from which other firms are able to adopt. This externality could be interpreted as a free rider problem, as firms have an incentive to wait before upgrading, and let other firms adopt first, in order to have a better chance of adopting a more productive technology.

Together with the static optimization problem, equations (10), (11), and (12) characterize optimal firm policies given equilibrium prices and a law of motion for the productivity distribution.

2.3. Adoption Costs

Technology adoption is costly. In our baseline specification, this cost takes the form of labor the firm must hire. The real cost of adoption, denoted by X, is

$$X(t) := \zeta \bar{L} \frac{W(t)}{P(t)}, \tag{13}$$

where the quantity of labor required to adopt a technology scales with population size and depends on the parameter $\zeta > 0$. The product of this quantity and the real wage determines the real cost of adoption. Note that the specification ensures that adoption cost grows in proportion to the real wage, and, thus, ensures the cost does not become increasingly small as the economy grows.

2.4. Entry, Exit, and the Mass of Varieties

There is a large pool of non-active firms that may enter the economy by paying an entry cost to gain a draw of an initial productivity from the same distribution from which adopters draw—the cross-section of incumbent productivities. We model the cost of entry as a multiple of the adoption cost for incumbents, $X(t)/\chi$, where $0 < \chi < 1$. Hence, χ is the ratio of adoption to entry costs. The restriction that $\chi \in (0, 1)$ means that incumbents have a lower cost of upgrading to a better technology than entrants have to start producing a new variety from scratch. Thus, the free entry condition is

$$X(t)/\chi = \int_{M(t)}^{\infty} V(Z, t) \mathrm{d}\Phi(Z, t), \tag{14}$$

which equates the cost of entry to the value of entry.

Exit occurs because firms die at an exogenous rate $\delta \geq 0$ that is independent of firm characteristics. For tractability, our theoretical results will focus on the limiting case in which there is no firm death ($\delta = 0$) and, hence, no entry.[10]

Even in the limiting case when ($\delta = 0$), the equilibrium number of varieties (firms), Ω, is endogenous and determined by the free entry condition. This allows us to study the growth and welfare implications of lowering trade costs on the endogenous number of varieties or "product

[10] In the Appendix, we solve the model with an arbitrary death rate $\delta \geq 0$.

innovation" as described in Atkeson and Burstein (2010).

3. Computing a Balanced Growth Path Equilibrium

In this section, we define and compute a Balanced Growth Path (BGP) equilibrium. Main results are then discussed in Sections 4-6. The Appendix documents the detailed steps involved in the computation of equilibrium and derivation of our main results.

3.1. Definition of a Balanced Growth Path Equilibrium.

Definition 1. *A Balanced Growth Path Equilibrium consists of an initial distribution $\Phi(0)$ with support $[M(0), \infty)$, a sequence of distributions $\{\Phi(Z,t)\}_{t=1}^{\infty}$, firm adoption and export policies $\left\{M(t), \hat{Z}(t)\right\}_{t=0}^{\infty}$, firm price and labor policies $\{p_d(Z,t), p_x(Z,t), \ell_d(Z,t), \ell_x(Z,t)\}_{t=0}^{\infty}$, wages $\{W(t)\}_{t=0}^{\infty}$, adoption costs $\{X(t)\}_{t=0}^{\infty}$, mass of varieties Ω, and a growth rate $g > 0$ such that:*

- *Given aggregate prices, costs, and distributions*
  - *$M(t)$ is the optimal adoption threshold and $V(Z,t)$ is the continuation value function*
  - *$\hat{Z}(t)$ is the optimal export threshold*
  - *Ω is consistent with free entry*
  - *$p_d(Z,t)$, $p_x(Z,t)$, $\ell_d(Z,t)$, and $\ell_x(Z,t)$ are the optimal firm static policies*
- *The gross value of adoption and entry equal $\int_{M(t)}^{\infty} V(Z',t)\phi(Z',t)dZ'$*
- *Markets clear at each date t*
- *Aggregate expenditure is stationary when re-scaled: $Y(0) = Y(t)e^{-gt}$*
- *The distribution of productivities is stationary when re-scaled:*

$$\phi(Z,t) = e^{-gt}\phi(Ze^{-gt},0)\ \forall\, t,\ Z \geq\ M(0)e^{gt}$$

In order to compute an equilibrium, we proceed in three steps. First, we derive the law of motion for the productivity distribution given a technology adoption policy $M(t)$. Second, given the law of motion, we solve for the firms' value function and adoption policy. Third, we solve for the growth rate g that ensures consistency between the first two steps.

3.2. The Productivity Distribution

This first step in computing the equilibrium takes the technology adoption policy as given and derives the law of motion for the productivity distribution. Below we highlight the key elements. Appendix B provides a complete technical derivation.

First, note that the minimum of the support of the productivity distribution is the adoption reservation productivity $M(t)$. Recall, that when adopting, a firm receives a random productivity draw from the distribution of producers. Except, perhaps, at time 0, the probability of drawing the productivity of a fellow adopting firm is infinitesimal. Therefore, firms adopting at time t will adopt a technology above $M(t)$ almost certainly. Thus, $M(t)$ is like an absorbing barrier sweeping through the distribution from below and, thus, is the minimum of the support.

The Kolmogorov Forward Equation (KFE) describes the evolution of the productivity distribution for productivities above the minimum of the support. The KFE is simply the the flow of adopters times the density they are redistributed into:

$$\frac{\partial \phi(Z,t)}{\partial t} = \underbrace{M'(t)\phi(M(t),t)}_{\text{flow of adopters}} \quad \times \quad \underbrace{\phi(Z,t)}_{\text{redistribution density}} \quad . \tag{15}$$

The flow of adopters is determined by the rate at which the adoption threshold sweeps across the density, $M'(t)$, and the amount firms the adoption boundary collects as the adoption boundary sweeps across the density from below, $\phi(M(t),t)$. Thus, the flow of adopters is

$$S(t) := M'(t)\phi(M(t),t). \tag{16}$$

Two features of the environment determine the density the adopters are redistributed into. Since adopters end up above $M(t)$ almost certainly and $M(t)$ is the lower support of the density, then the adopters are redistributed across the entire support of $\phi(Z,t)$. Since adopters draw directly from the productivity density, they are redistributed throughout the distribution in proportion to it. Thus, the redistribution density is $\phi(Z,t)$.

The KFE for the productivity distribution in equation (15) is an ordinary differential equation (ODE) in time. The solution to this ODE characterizes the productivity distribution at any date

$$\phi(Z,t) = \frac{\phi(Z,0)}{1 - \Phi(M(t),0)}. \tag{17}$$

That is, the distribution at date t is a truncation of the initial distribution at the minimum of support at time t, $M(t)$. The solution in equation (17) is rather general. It holds independent of the type of the initial distribution and independent of whether the economy is on a balanced growth path.

To maintain tractability in the static firm problem and solve for a balanced growth path, we assume that the initial distribution is Pareto

$$\Phi(Z,0) = 1 - \left(\frac{M(0)}{Z}\right)^{\theta}, \text{ with density, } \phi(Z,0) = \theta M(0)^{\theta} Z^{-\theta-1}, \tag{18}$$

where θ is the shape parameter and $M(0)$ is the initial minimum of support. This assumption has been exploited in similar models such as Kortum (1997), Jones (2005), and Perla and Tonetti (2014). Combining this distribution with the solution to the Kolmogorov Forward Equation in equation (17) is powerful. Together, they imply that the productivity distribution always remains Pareto with shape θ. Specifically, the density at any date t is

$$\phi(Z,t) = \theta M(t)^{\theta} Z^{-\theta-1}, \tag{19}$$

with flow of adopters

$$S = \theta g(t). \tag{20}$$

This common distributional assumption facilitates a solution in two ways. On the static dimension, it allows us to compute the equilibrium relationships, for all time, as one would in a variant of Melitz (2003). On the dynamic dimension, if the technology adoption policy is stationary when re-scaled, then this distribution satisfies the final requirement in Definition 1 that the distribution of productivities is stationary when re-scaled. Thus, it provides us an opportunity to find a balanced growth path.

Perla and Tonetti (2014) provides a complete discussion on why an initial distribution with a Pareto tail is necessary to support long run growth and why the Pareto distribution is the only initial condition consistent with the balanced growth path law of motion for the productivity distribution. The key reason is that power laws have a scale invariance property, which means that as the economy grows geometrically, the distribution's shape remains constant. Economically, that the tail does not get thinner over time means that there always remain enough better technologies available for adoption to incentivize sustained investment in adoption in the long run.

These restrictions on the initial productivity distribution are not as limiting as they might seem. In the Appendix, we solve an extended version of the model where firms receive exogenous productivity shocks which evolve according to a geometric Brownian motion. In this model, starting from *any* initial distribution—including those with finite support or even degenerate distributions of ex-ante identical firms—the stationary distribution is Pareto. For related results and further discussion see Luttmer (2012) and Benhabib, Perla, and Tonetti (2015).

3.3. Static and Dynamic Equilibrium Relationships

The second step in computing the equilibrium is to characterize the firm's value function and adoption policy, given the law of motion described above. To do so, we first to normalize the economy and make it stationary. We then describe the important static and dynamic equilibrium relationships on which the firm value function and adoption policy depend.

Normalization. We normalize the economy to be stationary. Roughly speaking, we do this by normalizing all variables by the endogenous reservation productivity threshold $M(t)$; Appendices C and D provide the complete details. Regarding notation, all normalized variables are lower case versions of the relationships described above. For example, lower case z represents $Z/M(t)$, i.e., a firm's productivity relative to the reservation productivity threshold. The normalized productivity distribution relative to the adoption threshold is constant over time, due to the Pareto initial condition:

$$\begin{aligned} f(z,t) &= M(t)\phi(zM(t),t) \\ f(z) &= \theta z^{-\theta-1} \end{aligned} \tag{21}$$

Static Equilibrium Relationships. There are four important static equilibrium relationships that we use repeatedly throughout the rest of the paper. Specifically, the common component to firms' profits, the export productivity threshold, average profits, and the home trade share.

Normalized profits of a firm are

$$\begin{aligned} \pi(z) &= \bar{\pi}_{\min} z^{\sigma-1} + (N-1)\left(\bar{\pi}_{\min} d^{1-\sigma} z^{\sigma-1} - \kappa\right) && \text{if } z \geq \hat{z} \\ \pi(z) &= \bar{\pi}_{\min} z^{\sigma-1} && \text{otherwise.} \end{aligned} \tag{22}$$

The common component of firms' profits, defined as $\bar{\pi}_{\min}$, is important for two reasons. First, the value $\bar{\pi}_{\min}$ represents the profits of the *marginal adopting firm*. Given our normalization where z is defined relative to the reservation productivity threshold, a firm with $z = 1$ is the firm that is on the margin between adopting and not. Since, by definition, the choices of the marginal firm determine the adoption decision, how $\bar{\pi}_{\min}$ changes with trade barriers is important to understanding how the incentives to adopt change.

Second, because $\bar{\pi}_{\min}$ is common to all firms, it summarizes how changes to trade barriers affect profits of all firms on the intensive margin. That is holding fixed firms' exporter status, it determines the benefit (or loss) to all firms from opening to trade.

The export productivity threshold, $\hat{z}$, in equation (22) is the productivity level at which a firm is just indifferent between entering an export market or selling only domestically. This export threshold can be expressed as

$$\hat{z} = d \left(\frac{\kappa}{\bar{\pi}_{\min}} \right)^{\frac{1}{\sigma-1}}, \tag{23}$$

which depends on variable trade costs, fixed trade costs, and the common component of profits.

The profits of the marginal firm and the exporter threshold allow us to compute two summary statistics that are useful in characterizing the relationship between growth, trade, and welfare. The first is the ratio of average profits relative to minimum profits $\bar{\pi}_{\min}$

$$\bar{\pi}_{\text{rat}} := \frac{1}{\bar{\pi}_{\min}} \int_{1}^{\infty} \pi(z) f(z) dz, \tag{24}$$

which integrates over the normalized profit levels (equation 22) according to the density (equation 21). As we show below, this summary statistic of the profit distribution summarizes the key trade-off for the marginal firm deciding to adopt a new technology and, thus, dictates the rate of economic growth.

The second statistic is a country's home trade share. This is the amount of expenditure a country spends on domestically produced varieties.

$$\lambda_{ii} := \frac{\bar{\pi}_{\min}}{\bar{\pi}_{\min} + (N-1)\hat{z}^{-\theta}\kappa}. \tag{25}$$

This relationship is important because this value summarizes the volume of trade in the economy and, thus, λ_{ii}^{-1} is a measure of openness. The home trade share connects with the profit ratio in equation (24) to provide a connection between growth and the observed volume of trade.

Dynamic Equilibrium Relationships. On the balanced growth path, the normalized continuation value function, value matching condition, and smooth pasting condition in equations (10–12) simplify to

$$(r-g)v(z) = \pi(z) - gz\frac{\partial v(z)}{\partial z}, \tag{26}$$

$$v(1) = \int_{1}^{\infty} v(z) f(z) \mathrm{d}z - \zeta, \tag{27}$$

$$\frac{\partial v(1)}{\partial z} = 0. \tag{28}$$

The major advantage of the normalized system is that it reduces the value function to one of z alone, removing the dependence on time. This mirrors the normalization of the productivity distribution. Thus, computing an equilibrium using the normalized system of equations involves solving an ordinary, as opposed to partial, differential equation.

The final, normalized dynamic equilibrium relationship is the free entry condition

$$\frac{\zeta}{\chi} = \int_1^\infty v(z) f(z) \mathrm{d}z, \tag{29}$$

which relates the normalized entry cost to the gross value of entry (and adoption).

3.4. Algorithm for Computing the Equilibrium

Given the law of motion for the productivity distribution and the normalized static and dynamic equilibrium relationships, we now outline how to solve for the equilibrium growth rate, with details in Appendices E and G.

We first solve the ordinary differential equation describing the firm's value function in equation (26) through the method of undetermined coefficients, using profits and the exporter threshold from equations (22) and (23) and ensuring that the smooth pasting condition in equation (28) is satisfied. The value function depends on a firm's productivity z, the common profit component $\bar{\pi}_{\min}$, the export threshold $\hat{z}$, and the rate of economic growth g.

We then insert this value function into the value matching condition (equation 27) which, when combined with the free entry condition (equation 29), yields the growth rate g as a function of $\hat{z}$ and $\bar{\pi}_{\min}$. Because the continuation value function of the marginal firm and the expected value of adoption both depend on the rate of economic growth, this boils down to finding a growth rate that makes the marginal firm indifferent between continuing to operate and adopting a new technology. Finally, using the free entry condition, $\bar{\pi}_{\min}$ and $\hat{z}$ can be solved for analytically, yielding g and all other equilibrium objects in closed form.

4. Growth and Trade

4.1. Growth and Trade

Proposition 1 provides the equilibrium growth rate as a function of parameters, completing the characterization of economic growth in a model with equilibrium technology diffusion, entry and exit, and selection into exporting.

Proposition 1 (Growth on the BGP). *If $\theta > \sigma - 1 > \theta\chi > 0$, then there exists a unique Balanced Growth Path Equilibrium with finite and positive growth rate*

$$g = \frac{\rho(1-\chi)}{\chi\theta}\bar{\pi}_{rat} - \frac{\rho}{\chi\theta}, \tag{30}$$

where the ratio of average profits to minimum profits is

$$\bar{\pi}_{rat} = \frac{\left(\theta + (N-1)(\sigma-1)d^{-\theta}\left(\frac{\kappa}{\zeta}\frac{\chi}{\rho(1-\chi)}\right)^{1-\frac{\theta}{\sigma-1}}\right)}{(1+\theta-\sigma)}. \tag{31}$$

Proof. See Appendix G. □

The parameter restrictions are twofold. First, for growth to be positive incumbents must be upgrading their technology. This requires that the relative entry cost be "large enough" (small χ) relative to the adoption cost, with the bound on the relative entry costs being the ratio of the demand elasticity parameter minus one relative to the parameter controlling productivity heterogeneity across firms. Second, as is standard, the love for variety as measured by the elasticity of substitution across goods can not be too large relative the to tail parameter that indexes heterogeneity in firm productivity. We impose these parameter restrictions throughout the rest of the paper.

The most interesting feature of Proposition 1 is that the growth rate is an affine function of the ratio of profits between the average and marginal firm. This profit ratio is the key summary statistic in this model and its sensitivity to trade costs will drive many of our main results. The intuition for why the growth rate is a function of the profit ratio is that the incentive to adopt depends on two competing forces: the expected benefit of a new productivity draw and the cost of taking that draw. The opportunity cost of adopting a better technology is the forgone profits from producing with the current technology. The expected benefit relates to the profits that the average new technology would yield. Proposition 1 tells us that a larger spread in the expected benefit relative to the opportunity cost increases the incentives to adopt and, thus, leads to faster economic growth.[11]

We can go one step further and connect the profit ratio to a country's home trade share. This establishes a connection between economic growth and the volume of trade. After some sub-

[11] This result is closely related to Hornstein, Krusell, and Violante (2011). In a McCall (1970) labor-search model they establish a relationship between the frequency of search and a summary statistic of wage dispersion—the ratio of the average wage to the minimum wage. In our model, growth is related to the frequency of firms searching to adopt new technologies and equation (30) shows how this depends on the ratio of profits between the average and marginal firm; similar to Hornstein, Krusell, and Violante (2011).

stitution, a country's home trade share in terms of primitives is

$$\lambda_{ii} = \frac{1}{1 + (N-1)d^{-\theta}\left(\frac{\kappa\chi}{\zeta\rho(1-\chi)}\right)^{1-\frac{\theta}{\sigma-1}}}. \tag{32}$$

Comparing equation (32) and (31) reveals that the home trade share tightly relates to the profit ratio. This connection allows us summarize growth as a function of the inverse of a country's home trade share in Corollary 1.

Corollary 1 (**Growth and the Volume of Trade**). *On the Balanced Growth Path, the relationship between growth and the volume of trade is*

$$g = \frac{\rho(1-\chi)}{\chi\theta}\frac{\sigma-1}{\theta-\sigma+1}\lambda_{ii}^{-1} - \frac{\rho}{\chi}, \tag{33}$$

with a country's home trade share given in equation (32).

Proof. See Appendix G. □

Corollary 1 relates to the results in Arkolakis, Costinot, and Rodriguez-Clare (2012) which connects the *level* of real wages to a countries home trade share. An interpretation of their results in the Melitz (2003) model is that the trade induced welfare gains from reallocation are completely summarized by the change in a country's home trade share. In our model, Proposition 1 tells us that the incentives to adopt new technologies are driven by the spread in profits between the average and the marginal firm. Similar to Arkolakis, Costinot, and Rodriguez-Clare's (2012) findings, Corollary 1 says that these distributional effects are summarized by the aggregate volume of trade.

How do changes in trade costs affect growth? A quick examination of equations (31) or (32) shows that a decrease in variable trade costs will increase the spread in profits between the average and marginal firm, reduce a country's home trade share, and increase the rate of economic growth. Proposition 2 summarizes these effects in the form of elasticities for a world-wide reduction in variable trade costs. Moreover, we provide a "sufficient statistic" representation of these elasticities that depends only on several parameters and observable trade statistics.

Proposition 2 (**Comparative Statics: Trade, Profits, and Growth**). *A decrease in variable trade costs…*

1. *Decreases a country's home trade share.*

$$\varepsilon_{\lambda_{ii},d} \;:=\; \frac{\partial \log \lambda_{ii}(d)}{\partial \log d} \;=\; \theta(1-\lambda_{ii}) \;>\; 0. \tag{34}$$

2. *Increases the spread in profits between the average and marginal firm.*

$$\varepsilon_{\bar{\pi}_{rat},d} \;:=\; \frac{\partial \log \bar{\pi}_{rat}(d)}{\partial \log d} \;=\; \frac{-(\sigma-1)\varepsilon_{\lambda_{ii},d}}{1+\lambda_{ii}(\theta-1)} \;<\; 0. \tag{35}$$

3. *Increases economic growth*

$$\varepsilon_{g,d} \;:=\; \frac{\partial \log g(d)}{\partial \log d} \;=\; \left(\frac{\chi(1+\theta-\sigma)}{(\sigma-1)(1-\chi)}\lambda_{ii}-1\right)^{-1}\varepsilon_{\lambda_{ii},d} \;<\; 0, \tag{36}$$

Proof. See Appendix G. □

The first result defines what we will call the "trade elasticity" $\varepsilon_{\lambda_{ii},d}$, which is the amount the home trade share declines (and volume of trade increases) as variable trade costs decline.[12] This elasticity takes a simple form which depends on the volume of trade and the parameter θ. As we discuss below, this is an important input into every other elasticity.

The second result connects the trade elasticity with reallocation effects. A reduction in trade costs increases the difference in profits between the average firm and the marginal firm. And the spreading of profits is tied to the trade elasticity. The basic idea is that lower trade costs induce high productivity exporting firms to expand and low productivity firms to contract as competition from foreign firms reduce their profits. Given that these distributional effects are summarized by the home trade share (Corollary 1), the change in the profit spread is proportional to the trade elasticity.

The third result in Proposition 2 shows that reductions in trade costs increase economic growth. The sensitivity of growth to changes in trade costs crucially depends on the trade elasticity. The intuition for this connection lies in the previous result. The reallocation effects from reductions in trade costs incentivizes more frequent technology adoption by both lowering the opportunity cost of adoption and increasing the expected benefit. For low productivity (non-exiting firms), the loss of market share and profits reduces the opportunity cost of adoption. Because exporting now has a larger return, this increases the expected benefit of a new technology. Together this leads firms to more frequently improve their technology. Because the frequency at which firms upgrade their technology is intimately tied to aggregate growth, growth increases as variable trade costs decrease.

The sufficient statistic representation of these elasticities isolates which parameters are important in quantifying how growth responds to reductions in trade costs. In particular, it shows

[12] We are abusing conventional terminology here. Typically the trade elasticity discussed in the literature is the log change in bilateral trade with respect to a log change in bilateral trade costs—and typically is just θ. Because of how it shows up everywhere else in our computations, we are computing the elasticity of the home-trade share, which we are calling the "trade elasticity."

that the important factors are the: level of trade flows λ_{ii}, extent of firm heterogeneity θ, curvature on the demand for varieties σ, and size of adoption costs relative to entry costs χ. The trade share is observed. There is an array of estimates in the literature for θ and σ. The only difficult parameter to discipline is χ. These observations shape our calibration strategy in Section 6.

This representation also illuminates the role of scale in our economy. First, notice that in both the growth rate and elasticity formulas, population size does not appear at all. This is largely by construction as the fixed costs of exporting and adoption are scaled by population. Second, examination of equations (30) and (31) in Proposition 1 shows that the number of countries in the economy affects economic growth. What Corollary 1 and Proposition 2 show, however, is that what ultimately matters is the volume of trade, and not the number of countries or the trade costs per se. As our discussion of market size effects in Section 5 makes clear, growth depends on the ratio of profits between the average and marginal firm—not the level of profits. That is, controlling for the amount of trade as summarized by λ_{ii}, the number of countries does not affect the relative benefit of adoption and, hence, the incentives of firms to adopt a new technology.

Our model has the prediction that changes in trade costs leads to permanent growth effects. The empirical evidence is inconclusive as to whether there are growth or level effects associated with changes in trade costs.[13] We view our main contribution as providing a better understanding of how trade affects the relative benefits of technology adoption across firms. This mechanism would be present independent of whether the economy was modeled in an endogenous or semi-endogenous growth framework.

4.2. Firms, Trade, and Technology Adoption

Below we focus on a firm's adoption policy and how it relates to the aggregate environment. Proposition 3 summarizes our results.

Proposition 3 (Firms and Technology Adoption). *Given an aggregate growth rate g ...*

1. *The time $\tau(z)$ until an individual firm with productivity z adopts a new technology is*

$$\tau(z) = \frac{\log(z)}{g}. \tag{37}$$

[13]We would also caution against using evidence from cross-country regressions (e.g., Frankel and Romer, 1999; Dollar and Kraay, 2004) to make inferences about our results because our results imply a non-linear relationship between growth and the volume of trade in Corollary 1, are in the context of a symmetric country equilibrium, and abstract from important mechanisms such as cross-country technology diffusion.

2. *The average time until adoption is*

$$\bar{\tau} = \frac{1}{\theta g}. \tag{38}$$

3. *Over a Δ length of time, the mass of firms that adopt is*

$$S\Delta = \frac{\Delta}{\bar{\tau}}. \tag{39}$$

Proof. See Appendix G. ☐

The first part of the proposition focuses on an individual firm and computes the time until it changes its technology. The second and third part of the proposition aggregate. Across all firms, the average time until adoption depends inversely on the growth rate and the Pareto shape parameter. Over an increment of time, the number of firms adopting is the flow of adopters times the length of time, which turns out to take a very simple form: the time increment multiplied by the inverse of the average time until adoption.

From a firm's perspective, more rapid economic growth means that it optimally waits a shorter amount of time before upgrading its technology. This effect of faster growth holds for all firms and, thus, the average time across all firms is shorter. Furthermore, this result implies that over a given time increment, more firms adopt.

Connecting these observations with the aggregate growth effects of trade in Proposition 2 we have predictions about firms' responses to a trade liberalization. Proposition 3 predicts that we should see more firms adopting new technologies in an open economy relative to a closed economy. More specifically, average, within-firm productivity growth is larger in the open economy relative to a closed economy.

These results connect with the motivating evidence discussed in Section 1.2. The predictions in Proposition 3 imply that an empirical specification which projects changes in firm level measures of technology on measures of openness should display a positive relationship. This is exactly what empirical papers using specifications of this type find in the data (see, e.g., Pavcnik, 2002; Topalova and Khandelwal, 2011; Bloom, Draca, and Reenen, 2015).

Our model makes distinct predictions about who adopts. In our model, average, within-firm productivity growth hides heterogeneity in who adopts and who does not. In particular, firms at the top of the productivity distribution continue to operate with their existing technology while firms at the bottom adopt and experience productivity growth. In contrast, in Atkeson and Burstein (2010), it is the large, exporting firms that innovate more; small import-competing firms decrease their innovation.

Our model's predictions for who adopts has empirical support. An empirical specification which interacts the openness measure with a firm's initial productivity level should find that firms with the lowest initial productivity levels increase their productivity the most. Using firm-level data from Spain, Steinwender (2014) finds heterogeneous responses with the least productive, import-competing firms increasing their productivity the most.

A second piece of support for heterogeneous productivity responses is in Lileeva and Trefler (2010). They found that new Canadian exporters induced by the Canada-US Free Trade Agreement (which came from all parts of the productivity distribution) experienced productivity gains while existing exporters' productivity did not improve. Our model predicts the same pattern in that existing exporters would experience no productivity growth, but new exporters would.

5. Reallocation vs. Market Size Effects

Heterogeneity in firms' incentives and actions are the essential ingredients driving the relationship between trade and growth in our model. The heterogeneous incentives induce a reallocation effect across firms that is distinct from the "market size" mechanisms emphasized in the previous literature, i.e., the ability to spread the same cost of adoption across a larger market resulting in growth effects from openness. In this section we highlight the key mechanism active in our model by removing the type of heterogeneity that links trade and growth.

To focus on traditional market size forces and abstract from the role of distributional effects, we set the fixed cost of exporting equal to zero and keep variable trade costs low enough to study an environment in which all firms export. Equilibrium objects in this environment are labeled with a superscript k, since this model resembles a heterogeneous firm version of Krugman (1980). Proposition 4 summarizes growth in this model.

Proposition 4 (Growth with No Selection into Exporting). *In the model with $\kappa = 0$ and all firms selling internationally, the growth rate is*

$$g^k = \frac{\rho(1-\chi)}{\chi\theta}\bar{\pi}^k_{rat} - \frac{\rho}{\chi\theta}, \tag{40}$$

where the ratio of average profits to minimum profits is

$$\bar{\pi}^k_{rat} = \frac{(1+(N-1)d^{1-\sigma})\bar{\pi}_{\min}E[z^{\sigma-1}]}{(1+(N-1)d^{1-\sigma})\bar{\pi}_{\min}} = \frac{\theta}{1+\theta-\sigma}. \tag{41}$$

Proof. See Appendix F. □

The expression for the growth rate in equation (40) takes the same form as that in Proposition 1: Growth is an affine function of the ratio of profits between the average firm and the marginal firm. Even though the details of international trade differ, the technology adoption choice is the same and, hence, the aggregate growth rate takes the same form.

The only difference between Proposition 4 and Proposition 1 is that the profit ratio is now independent of trade costs. This implies that growth is independent of trade costs. When there is no selection and all firms export, reductions in trade costs do not induce reallocation—all firms' profits increase by the same proportion in response to lower trade costs.

The intuition for this result is the following: Technology adoption in our model depends on a comparison between the expected value of adoption versus the value of continuing to operate with the existing technology. Lower trade costs have two effects on a firm's incentive to adopt a new technology. Just as in the model with selection into exporting, lower trade costs increase the expected value of adopting a new technology. However, when all firms export, lower trade costs also increase the value of continuing to operate the old technology for the marginal firm. The profit ratio summarizes this comparison. Because all firms' profits scale up by the same proportion, lower trade costs do not affect the *relative* benefit of adoption.[14] Thus, changes in trade costs do not change the rate of economic growth.

In contrast to this neutrality result, with selection into exporting, the differential exposure of firms to trade opportunities affects the distribution of profits and, hence, the incentives to adopt and economic growth. To illustrate these mechanics, we decompose the profit ratio in equation (31) from Proposition 1 in our baseline model in the following way:

$$\bar{\pi}_{\text{rat}} = \underbrace{\frac{\theta}{1+\theta-\sigma}}_{=\bar{\pi}^{k}_{\text{rat}}} + \frac{(N-1)(\sigma-1)d^{-\theta}\left(\frac{\kappa}{\zeta}\frac{\chi}{\rho(1-\chi)}\right)^{1-\frac{\theta}{\sigma-1}}}{1+\theta-\sigma}. \tag{42}$$

The profit ratio in our baseline economy is now written as the sum of two terms. The first term is the same as the profit ratio above in the No Selection into Exporting economy. The second term isolates the affects from trade-induced reallocation. This second term shows how reductions in either variable or fixed trade costs "spread" the distribution of profits relative to the No Selection into Exporting economy, incentivize faster adoption, and lead to increases in economic growth.

[14] This result is similar to one in Eaton and Kortum (2001) who highlight that there are two competing forces from a larger market: a larger market makes an innovation more valuable, but also makes it more costly to achieve a new innovation. In their model, these two mechanisms cancel out leaving a result similar to Proposition 4.

6. Welfare

This section studies the welfare effects of lower barriers to trade. First we document analytically how changes in trade costs affect the initial level of consumption and how consumption and growth rate effects combine to determine welfare. Then we quantify the relationship between trade costs, growth, consumption, and welfare. We calibrate the model and calculate the differences between the baseline equilibrium and the autarky equilibrium to quantify the welfare gains from trade.

6.1. Consumption and Welfare: Qualitative Analysis

How does welfare change with trade costs? While increased trade leads to faster economic growth, we show that there are more subtle effects of opening to trade on welfare. In particular, the gains from trade are a race between the positive dynamic growth effects, the positive static reallocation effects, and the negative static effects of less varieties consumed and reallocation of workers away from goods production to adoption activity .

Time zero utility and the associated initial level of consumption are

$$\bar{U} = \rho \log(c) + g, \tag{43}$$

$$c = \left(1 - \tilde{L}\right) \Omega^{\frac{1}{\sigma-1}} \lambda_{ii}^{\frac{1}{1-\sigma}} \left(\mathbb{E}\left[z^{\sigma-1}\right]\right)^{\frac{1}{\sigma-1}}. \tag{44}$$

The level of consumption depends on several factors. The $(1 - \tilde{L})$ term is the amount of labor devoted to the production of goods—as opposed to adoption and entry activity. The second term is the mass of varieties. The third term is the home trade share and the fourth term is just the $\sigma - 1$ moment of the productivity distribution. Again, recall from the discussion of Corollary 1, that the trade share is a summary statistic for the distribution of activity across producers.

With changes in trade costs, the initial level of consumption changes for several reasons: changes in the home trade share, the mass of varieties, and the share of labor engaged in goods production. Proposition 2 showed how the home trade share changes. The proposition below formalizes how the mass of varieties and the share of labor engaged in goods production changes with trade costs.

Proposition 5 (Comparative Statics: Variety and Labor Allocations). *A decrease in variable trade costs...*

1. *Reduces domestic varieties.*

$$\varepsilon_{\Omega,d} \ := \ \frac{\partial \log \Omega(d)}{\partial \log d} \ = \ \left(1 - \frac{1+\theta-\sigma}{\theta\sigma(1-\chi)}\lambda_{ii}\right)^{-1} \varepsilon_{\lambda_{ii},d} \ > \ 0. \tag{45}$$

2. *Reduces the share of workers in goods production.*

$$\varepsilon_{\tilde{L},d} \ := \ \frac{\partial \log(1-\tilde{L}(d))}{\partial \log d} \ = \ \left(\frac{\theta\sigma(1-\chi)}{1+\theta-\sigma}\lambda_{ii}^{-1} - 1\right)^{-1} \varepsilon_{\lambda_{ii},d} \ > \ 0. \tag{46}$$

Proof. See Appendix G. □

Lower trade costs decrease the mass of domestic varieties produced. This is consistent with the Melitz (2003) logic that reductions in trade costs induce exit. Lower trade costs also result in less labor allocated to the production of goods and more labor allocated to the fixed costs of adoption and exporting. In a sense, there is more "investment" in the adoption of technology and exporting—a force towards less consumption.

Proposition 6 (Consumption Effects). *The change in initial consumption from a change in trade costs is*

$$\varepsilon_{c,d} = \varepsilon_{\tilde{L},d} + \frac{\varepsilon_{\Omega,d} - \varepsilon_{\lambda_{ii},d}}{\sigma - 1} > 0. \tag{47}$$

Proof. See Appendix G. □

Proposition 6 shows that there are two drags on the initial level of consumption and one offsetting gain. Reductions in trade costs reallocate labor away from the production of goods and, hence, the consumption of goods ($\varepsilon_{\tilde{L},d} > 0$). Reductions in trade costs lead to less production of domestic varieties ($\varepsilon_{\Omega,d} > 0$), which reduces consumption due to the love for variety CES final goods aggregator. Counteracting these forces are gains from an increase in the mass of foreign varieties consumed and that these foreign firms are relatively highly productive. These forces are summarized by a decrease in the home trade share ($\varepsilon_{\lambda_{ii},d} > 0$).

Which of these forces wins? The initial consumption level decreases with lower trade costs. To see this, note that from equation (45), the loss in domestic varieties is larger than the gain from importing foreign goods ($\varepsilon_{\Omega,d} > \varepsilon_{\lambda_{ii},d}$). Given that labor is always reallocated away from production as trade costs decrease, equation (47) shows that the level of initial consumption will decrease with lower trade costs.

This result deserves some discussion. First, while the initial level of consumption decreases, there is an inter-temporal gain as consumption in future periods is higher (Proposition 2).

Second, our model's static gains from trade need not correspond with those in purely static models of trade. In Melitz (2003), the typical parameterization finds that domestic variety falls, but the gains from foreign variety more than compensate for this loss.[15] In our model, dynamics lead to additional effects on the free-entry condition. Wages increase due to increased demand for labor, which increases the cost of entry. However, the expected value of entry does not increase in equal proportion, as profits are reallocated across firms and these profits are discounted at a higher rate because of faster growth. Thus, to satisfy the free-entry condition the reduction in domestic variety is larger (relative to static models). Atkeson and Burstein (2010) have a similar quantitative finding that the gains due to increased innovation by firms are offset by losses in variety and reallocation of resources away from production for consumption.

Finally, we should note that the race between these welfare reducing and welfare enhancing effects from trade cost changes is an important normative feature of our model relative to other idea-flow models studying trade and growth. For example, in Alvarez, Buera, and Lucas (2014), there are no resource costs associated with acquiring ideas. As a result, when trade facilitates faster idea acquisition and economic growth, there is no corresponding increase in costs. In contrast, in our model, faster idea acquisition comes with the cost of more labor being allocated away from production.

Using equations (43) and (44) and the results in Proposition 2, 5 and 6, the elasticity of utility with respect to a change in trade costs—the welfare gain from trade—is summarized below in Proposition 7.

Proposition 7 (**Welfare Effects**). *The welfare change from a change in trade costs is*

$$\varepsilon_{\bar{U},d} = \frac{\rho^2}{\bar{U}}\,(\rho\,\varepsilon_{c,d} + g\,\varepsilon_{g,d}). \tag{48}$$

Proof. See Appendix G. □

The welfare gain from trade is proportional to the change in the initial level of consumption, $\varepsilon_{c,d}$, and the change in the growth rate, $\varepsilon_{g,d}$.[16] Proposition 2 tells us that the benefit from growth is positive and Proposition 6 shows that the effect on consumption negative. Even with the positive impact from increased growth, the effect of lower trade costs on welfare is ambiguous.

[15] As our phrasing suggests, this need not be the case. See the discussion on page 1713 of Melitz (2003).

[16] Note that since $d > 0$, the sign of the elasticity and the derivative of utility with respect to trade are equal if and only if $\bar{U} > 0$. With $\log$ utility the sign of $\bar{U}$ depends on initial conditions on the mean of the productivity distribution and the population size. Most importantly, the sign of the derivative of utility with respect to trade costs is independent of these initial conditions and the sign of utility.

The Appendix derives necessary and sufficient conditions to sign the welfare difference (see equation G.50). In principal, it is feasible that consumption decreases enough to overwhelm the gains from faster growth. This might seem puzzling, but when the reduction in consumption is large enough, these effects are largely driven by the decrease in variety. An interpretation of the loss in variety is a loss of ideas or knowledge as discussed in Atkeson and Burstein (2015). This does not, however, appear to the empirically relevant case. For all calibrations we have explored, the conditions are satisfied such that welfare is always higher with lower trade costs.

6.2. Consumption and Welfare with No Selection into Exporting

To further isolate the race between the competing welfare reducing and welfare enhancing effects, consider the welfare effects in our model when all firms export. Proposition 8 details how trade, growth, varieties, labor allocations, and welfare are affected by trade costs in the economy in which there is no selection into exporting.

Proposition 8 (Comparative Statics with No Selection into Exporting). *In the model with $\kappa = 0$ and all firms selling internationally, a decrease in variable trade costs...*

1. *Reduces the home trade share*

$$\varepsilon^k_{\lambda_{ii},d} \;=\; \frac{\partial \log \lambda^k_{ii}(d)}{\partial \log d} \;= (\sigma - 1)(1 - \lambda^k_{ii}) \;>\; 0, \tag{49}$$

where,

$$\lambda^k_{ii} = \left(1 + (N-1)d^{1-\sigma}\right)^{-1}. \tag{50}$$

2. *Does not change the growth rate, the mass of domestic varieties, or the share of workers in goods production*

$$\varepsilon^k_{g,d} \;= 0, \quad \varepsilon^k_{\Omega,d} \;= 0, \quad \varepsilon^k_{\tilde{L},d} \;= 0. \tag{51}$$

3. *Increases the initial level of consumption*

$$\varepsilon^k_{c,d} = -\frac{\varepsilon^k_{\lambda_{ii},d}}{\sigma - 1} < 0, \tag{52}$$

and increases welfare

$$\varepsilon^k_{\bar{U},d} = \frac{\rho^2}{\bar{U}}\, \rho\, \varepsilon^k_{c,d} < 0. \tag{53}$$

Proof. See Appendix F. □

Lower trade costs lower the home trade share, although now moderated by $(\sigma - 1)$ as opposed to θ. In addition to growth being independent of trade costs, the share of workers allocated to goods production does not change with trade costs either. Thus, while this model does not have dynamic gains from trade, it does not experience some of the losses associated with dynamics, such as labor reallocation.

The mass of varieties does not change either. The reason is that when all firms export, all firms' profits scale up by the same amount as the real wage and hence the entry cost. As trade costs decline, all profits increase by the same amount and the expected value of entry increases by this amount, but the cost of entry increases in the same proportion as well.[17] Thus, the mass of varieties need not change for the free-entry condition to be satisfied.

These observations help explain why growth effects are associated with a loss of variety in Proposition 5. When there is selection into exporting, in response to a decrease in trade costs, the normalized expected value of entry increases less than the cost of entry because of the reallocation of profits across firms. This implies that a reduction in varieties must occur for the free-entry condition to be satisfied. Thus, the same exact reallocation effects which lead to faster economic growth, generate the loss of varieties.

The final line of Proposition 8 computes the total gains from trade. These gains are purely static and these static level effects only operate through the component associated with the home trade share. This result is similar to the welfare gain calculations in Arkolakis, Costinot, and Rodriguez-Clare (2012). Unlike the finding in Arkolakis, Costinot, and Rodriguez-Clare (2012) that the gains from trade are equivalent in models with selection or without, a comparison of Proposition 7 to Proposition 8 shows that the essential element to delivering dynamic gains from trade is the reallocation effect introduced by selection into exporting.

6.3. Calibration

In the next sections we calibrate the model, quantify the welfare gains or losses from trade, and decompose them into consumption and growth components.

Our calibration strategy chooses parameters by using a mix of normalization, selections from the literature, and calibration to directly match key features of the data. We should note that given the sufficiency results described above, some of these parameters are only indirectly necessary to perform counterfactuals.

The normalizations and pre-selected parameters are described in the top panel of Table 1. We

[17] While the independence of the growth rate and trade costs is true independent of whether the cost of adoption is in labor or goods, that the elasticity of varieties is independent of trade costs relies on the cost of adoption being in labor only. This result is similar to Atkeson and Burstein's (2010) Proposition 2 that states a similar neutrality result on the mass of varieties when the research good in their model uses only labor.

Table 1: Calibration

Parameter	Value	Source or Target
Number of Countries, N	10	—
Technology Adoption Cost, ζ	1.0	Normalization
Curvature Parameters, σ	3.0	Broda and Weinstein (2006)
Discount Rate, ρ	0.02	—
Iceberg Trade Cost, d	4.04	23.2 percent aggregate trade share
Export Fixed Cost, κ	0.01	20 percent of establishments export
Pareto Shape Parameter, θ	3.18	Exporters' domestic shipments = 4.8 $\times$ non-exporters'
Entry Cost Relative to Adoption Cost $1/\chi$	2.18	Two percent growth rate

Note: The target moment for an import trade share of 23.2 comes from Simonovska and Waugh (2014). The targets regarding exporters are from Bernard, Eaton, Jensen, and Kortum (2003).

set the number of countries equal to ten. Per the implications of Corollary 1 and Proposition 2 this choice has no material impact on the answer to the question regarding how growth responds to trade. We also normalize the technology adoption cost to the value one.

The parameter σ controlling the elasticity of substitution across varieties is set equal to three. This value is near the median estimate of the elasticity of substitution across varieties from Broda and Weinstein (2006). The discount factor is set to two percent. Given that our target growth rate is two percent, this implies a real interest rate of four percent.

The bottom panel of Table 1 reports parameters (and results) that were picked to target moments in the data. Specifically, the iceberg cost, Pareto shape parameter, and fixed costs (of export and technology) were picked to target four moments about aggregate trade, exporters, and growth: (i) US manufacturing import share is 23.2 (Simonovska and Waugh, 2014), (ii) exporters' domestic shipments are 4.8 times larger than non-exporters' (Bernard, Eaton, Jensen, and Kortum, 2003), (iii) 20 percent of establishments export (Bernard, Eaton, Jensen, and Kortum, 2003) and (iv) a two percent growth rate. While all the parameters are jointly determined, we enumerate them with the moments in the data that help identify their values.

These moments were chosen on the basis of our theoretical results above. The home trade share and, thus, the import share is a summary statistic for the profit ratio and it shows up in all the elasticity formulas. Thus, it is natural to ask the model to match this moment. The Pareto shape parameter determines properties of the distribution of firms and the trade elasticity (equation 34). Our approach is to use one property of firm size—the size differential between exporters and non-exporters—and then check ex-post if the chosen parameter is consistent with

Welfare Gains From Trade

	Autarky	Baseline	Change (in %)
Growth	1.39	2.00	43.8
Initial Consumption Level	2.70	2.49	-7.78
Labor in Production	0.82	0.78	-4.88
Domestic Variety	2.00	1.71	-14.5
"ACR" Effect	1.00	1.14	14.0
Welfare	—	—	13.3
Consumption Equivalent	—	—	23.9

alternative estimates of the trade elasticity. The fraction of establishments that export largely determines the exporting fixed cost. The entry cost is then ultimately picked to ensure that the growth rate in the economy is two percent.

We find our estimate of the Pareto shape parameter to be consistent with several studies that focus on estimating these parameters from trade data or firm-level moments. In particular, the value of 3.18 is close to Simonovska and Waugh's (2014) point estimate of 3.6 for the Melitz (2003) model. Bernard, Eaton, Jensen, and Kortum (2003) and Eaton, Kortum, and Kramarz (2011) find similar values using similar firm-level moments. The similarity between our estimate and these previous estimates is good given the dual role that this parameter plays in determining both the trade elasticity and the properties of the firm-size distribution.

Finally, we find the relative size of the entry cost to be about twice the adoption cost. We have no prior on the plausibility or implausibility of this result.

6.4. Welfare Gains From Trade

Given the calibrated parameters in Table 1, we study how much growth, consumption, and welfare differ across an economy in autarky compared to one with the observed levels of trade.

The first row of Table 2 reports the difference in economic growth. Moving from autarky to observed levels of trade increases economic growth by a large amount, approximately two-thirds of a percentage point. To put this in context, this implies that after 100 years, the open economy will have nearly twice the income level as the closed economy.

The next four rows of Table 2 break down the change in consumption and its subcomponents:

the change in labor allocated to production, the change in varieties, and the change coming from an increase in the volume of trade (which we label for lack of a better term the "ACR" effect).[18]

Per the discussion of Proposition 6, the level of consumption is lower in the open economy. At our calibrated parameter values, the loss of varieties basically offsets the static gains from trade (ACR effect). Returning to the elasticity calculations in Proposition 5, this can be seen by examining the bracketed term in equation (45). With σ and θ both being about three, the term in brackets is just slightly less than one and, thus, the loss of variety offsets the ACR effect. Add in the reallocation of labor away from production and the level of consumption decreases.

The final two rows of Table 2 report the total change in welfare in percent terms and via a consumption equivalent. Including both the lower consumption level and the higher growth rate, we find welfare in an economy with the observed levels of trade compared to autarky is about 13 percent higher. The last row presents the consumption equivalent: the increase in initial consumption under autarky a consumer requires to make them indifferent between living in the autarky and observed trade equilibria. This works out be about 24 percent.

There are a couple of points of note regarding this welfare calculation. First, the 13 percent welfare gain is of the same magnitude as the standard static gains implied by the ACR effect. In this sense, there is little difference between the dynamic gains from trade and the gains from trade in a static trade model. As emphasized in the discussion above, the reason is that the dynamic gains from trade come with costs. Second, this welfare calculation only compares one balanced growth path equilibrium to another—transition dynamics are not included. Although very difficult to compute, we have reason to suspect that incorporating these transition dynamics would magnify the welfare effects. Since exit is not immediate, excess variety would be enjoyed along the transition.

7. Conclusion

This paper contributes a dynamic model of growth and international trade, driven by technology diffusion. Firms choose to upgrade their productivity through technology adoption to remain competitive and profitable. Highly productive firms benefit from a decline in trade costs, as they are the exporters who can take advantage of increased sales abroad. Low productivity firms only sell domestically and are hurt by the increased competition from foreign firms. The incentives to adopt, and thus the growth rate, are summarized by the ratio of profits between the average and the marginal adopting firm. In equilibrium, lower trade costs increase

[18] We are abusing terminology here. The result from Arkolakis, Costinot, and Rodriguez-Clare (2012) is that the static gains are completely summarized by the home trade share and the parameter θ. This is not case here. Our "ACR" effect only represents one component of the static gains.

this profit spread which leads lower productivity firms to upgrade their technology more frequently and increases aggregate growth.

In this model, the change in welfare from lower trade costs is a weighted sum of this increase in economic growth and a change in the initial level of consumption. The change in consumption is a sum of three components: a static gain from trade as in Arkolakis, Costinot, and Rodriguez-Clare (2012), a change in the mass of varieties consumed, and a change in the amount of labor allocated to the production of goods. We prove that opening to trade reduces the initial level of consumption and dampens the gains from faster economic growth. Our calibrated baseline model suggests, for a move from autarky to the observed volume of trade, that this domestic technology adoption mechanism leads to 0.60 percentage point higher economic growth and 13 percent higher welfare. This is nearly equivalent to the gains a static Arkolakis, Costinot, and Rodriguez-Clare (2012) measurement would deliver. The reason is that the dynamic gains from trade do not come for free. These trade-induced within-firm productivity improvements and their aggregate growth effects come with costs—and these costs take the form of losses in varieties produced and consumed and reallocation of resources away from goods production.

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Appendix (For Online Publication)

A. Environment and Optimization Problems

To demonstrate the extensibility of the model, in this appendix we derive conditions for a version of the model in which there is CRRA power utility, a weakly positive probability of death δ, costs of adoption and entry can be a convex combination of labor and goods, and exogenous shocks to TFP that follow a geometric Brownian motion with drift. For expositional clarity, the body of the paper studies the special case of log utility, no exogenous productivity shocks, costs of adoption and entry in labor only, and studies the limiting economy as the death rate, and thus the BGP equilibrium entry rate, is zero.

A.1. Consumer Preferences and Labor Supply

All countries are symmetric. In each country there exists a representative consumer of measure $\bar{L}$. The utility of the consumer is given by a constant relative risk aversion (CRRA) function in final goods consumption (C), given an inelastic supply of labor ($\bar{L}$). The coefficient of relative risk aversion is $\gamma \geq 0$, and the time discount rate is ρ.

Final goods are produced through CES aggregation of an endogenous number of intermediate varieties, including those produced domestically and those imported from abroad. There is an endogenous mass $\Omega(t)$ of firms operating in each country. The flow of intermediate firms adopting a new technology is $\Omega(t)S(t)$ and the flow of firms entering the market and creating a new variety is $\Omega(t)\delta(t)$. The consumer purchases the final consumption good, invests in technology adoption with a real cost of $X(t)$ per upgrading intermediate firm, and invests in firm entry with a real cost of $X(t)/\chi$.[19] Consumers income consists of labor earnings paid at wage $W(t)$ and profits from their ownership of the domestic firms. Aggregate profits from selling domestically are $\bar{\Pi}_d(t)$ and aggregate profits from exporting to the $N-1$ foreign countries is $(N-1)\bar{\Pi}_x(t)$). Thus, welfare at time $\tilde{t}$ is

$$\bar{U}(\tilde{t}) = \int_{\tilde{t}}^{\infty} U(C(t))e^{-\rho(t-\tilde{t})}\mathrm{d}t$$
$$\text{s.t. } C(t) + \Omega(t)X(t)(S(t) + \delta(t)/\chi) = \frac{W(t)}{P(t)}\bar{L} + \bar{\Pi}_d(t) + (N-1)\bar{\Pi}_x(t). \tag{A.1}$$

[19] Firms are maximizing real profits, discounting using the interest rate determined by the consumer's marginal rate of substitution. Hence, the investment choice of the consumer and firm is aligned, and consumers will finance upgrades to their existing firms through equity financing. As consumers own a perfectly diversified portfolio of domestic firms, they are only diluting their own equity with this financing method.

A.2. The Static Firm Problem

Intermediate Goods Demand. There is a mass $\Omega(t)$ of intermediate firms in each country that are monopolistically competitive, and the final goods sector is perfectly competitive. The final goods sector takes prices as given and aggregates intermediate goods with a CES production function, with $\sigma > 1$ the elasticity of substitution between all available products.

The CDF of the productivity distribution at time t is $\Phi(Z, t)$, normalized such that $\Phi(\infty, t) = 1$ for all t. Therefore, the total mass of firms with productivity below Z at time t is $\Omega(t)\Phi(Z, t)$.

Drop the t subscript for clarity. The standard solutions follow from maximizing the following final goods production problem,

$$\max_{Q_d, Q_x} \left[\Omega \int_M^\infty Q_d(Z)^{(\sigma-1)/\sigma} \mathrm{d}\Phi(Z) + (N-1)\Omega \int_{\hat{Z}}^\infty Q_x(Z)^{(\sigma-1)/\sigma} \mathrm{d}\Phi(Z) \right]^{\sigma/(\sigma-1)} \tag{A.2}$$

$$\text{s.t. } \Omega \int_M^\infty p_d(Z) Q_d(Z) \mathrm{d}\Phi(Z) + (N-1)\Omega \int_{\hat{Z}}^\infty p_x(Z) Q_x(Z) \mathrm{d}\Phi(Z) = Y. \tag{A.3}$$

Defining a price index P, the demand for each intermediate product is,

$$Q_d(Z) = \left(\tfrac{p_d(Z)}{P}\right)^{-\sigma} \tfrac{Y}{P}, Q_x(Z) = \left(\tfrac{p_x(Z)}{P}\right)^{-\sigma} \tfrac{Y}{P} \tag{A.4}$$

$$P^{1-\sigma} = \Omega \left(\int_M^\infty p_d(Z)^{1-\sigma} \mathrm{d}\Phi(Z) + (N-1) \int_{\hat{Z}}^\infty p_x(Z)^{1-\sigma} \mathrm{d}\Phi(Z) \right). \tag{A.5}$$

Static Profits. A monopolist operating domestically chooses each instant prices and labor demand to maximize profits, subject to the demand function given in equation (A.4),

$$P\Pi_d(Z) := \max_{p_d, \ell_d} \{ (p_d Z \ell_d - W \ell_d) \} \text{ s.t. equation (A.4)}. \tag{A.6}$$

Where $\Pi_d(Z)$ is the real profits from domestic production.

Firms face a fixed cost of exporting, $\kappa \geq 0$. To export, a firm must hire labor in the foreign country to gain access to foreign consumers. This fixed cost is paid in market wages, and is proportional to the number of consumers accessed. Additionally, exports are subject to a variable iceberg trade cost, $d \geq 1$, so that firm profits from exporting to a single country (i.e., export profits per market) are

$$P\Pi_x(Z) := \max_{p_x, \ell_x} \left\{ \left(p_x \tfrac{Z}{d} \ell_x - W \ell_x - \bar{L}\kappa W \right) \right\} \text{ s.t. equation (A.4)}. \tag{A.7}$$

Optimal firm policies consist of $p_d(Z)$, $p_x(Z)$, $\ell_d(Z)$, and $\ell_x(Z)$ and determine $\Pi_d(Z)$ and $\Pi_x(Z)$. As is standard, it is optimal for firms to charge a constant markup over marginal cost, $\bar{\sigma} :=$

$(\sigma - 1)/\sigma$.

$$p_d(Z) = \bar{\sigma}\tfrac{W}{Z}, \tag{A.8}$$

$$p_x(Z) = \bar{\sigma}d\tfrac{W}{Z}, \tag{A.9}$$

$$\ell_d(Z) = \tfrac{Q_d(Z)}{Z}, \quad \ell_x(Z) = d\tfrac{Q_x(Z)}{Z}. \tag{A.10}$$

To derive firm profits, take equation (A.6) and divide by P to get $\Pi_d(Z) = \frac{p_d(Z)}{P}Q_d(Z) - \frac{Q_d(Z)}{Z}\frac{W}{P}$. Substitute from equation (A.8), as $W = \frac{Zp_d(Z)}{\bar{\sigma}}$, to yield $\Pi_d(Z) = \frac{1}{\sigma}\frac{p_d(Z)}{P}Q_d(Z)$. Finally, use $Q_d(Z)$ from equation (A.4) to show

$$\Pi_d(Z) = \tfrac{1}{\sigma}\left(\tfrac{p_d(Z)}{P}\right)^{1-\sigma}\tfrac{Y}{P}. \tag{A.11}$$

Using similar techniques, export profits per market are

$$\Pi_x(Z) = \max\left\{0, \tfrac{1}{\sigma}\left(\tfrac{p_x(Z)}{P}\right)^{1-\sigma}\tfrac{Y}{P} - \bar{L}\kappa\tfrac{W}{P}\right\}. \tag{A.12}$$

Since there is a fixed cost to export, only firms with sufficiently high productivity will find it profitable to export. Solving equation (A.12) for the productivity that earns zero profits gives the export productivity threshold. That is, a firm will export iff $Z \geq \hat{Z}$, where $\hat{Z}$ satisfies

$$\left(\tfrac{p_x(\hat{Z})}{P}\right)^{1-\sigma} = \sigma\bar{L}\kappa\tfrac{W}{Y}, \tag{A.13}$$

$$\hat{Z} = \bar{\sigma}d(\sigma\bar{L}\kappa)^{\frac{1}{\sigma-1}}\left(\frac{W}{P}\right)\left(\frac{W}{Y}\right)^{\frac{1}{\sigma-1}}. \tag{A.14}$$

Let aggregate profits from domestic production be $\bar{\Pi}_d$ and aggregate export profits per market be $\bar{\Pi}_x$.

$$\bar{\Pi}_d := \Omega\int_M^\infty \Pi_d(Z)\mathrm{d}\Phi(Z). \tag{A.15}$$

$$\bar{\Pi}_x := \Omega\int_{\hat{Z}}^\infty \Pi_x(Z)\mathrm{d}\Phi(Z). \tag{A.16}$$

The trade share for a particular market, λ, is

$$\lambda = \Omega\int_{\hat{Z}}^\infty \left(\frac{p_x(Z)}{P}\right)^{1-\sigma}\mathrm{d}\Phi(Z). \tag{A.17}$$

A.3. Firms Dynamic Problem

Stochastic Process for Productivity Assume that operating firm's have (potentially) stochastic productivity following the stochastic differential equation for geometric Brownian motion (GBM),

$$\mathrm{d}Z_t/Z_t = (\mu + \upsilon^2/2)\mathrm{d}t + \upsilon \mathrm{d}W_t, \tag{A.18}$$

where $\mu \geq 0$ is related to the drift of the productivity process, $\upsilon \geq 0$ is the volatility, and W_t is standard Brownian motion.

At any instant in time, a firm will exit if hit by a death shock, which follows a Poisson process with arrival rate $\delta \geq 0$. Thus, all firms have the same probability of exiting and the probability of exiting is independent of time.

The case of $\mu = \upsilon = \delta = 0$, is the baseline model studied in the body of the paper.

Firm's Problem. Define a firm's total real profits as

$$\Pi(Z,t) := \Pi_d(Z,t) + (N-1)\Pi_x(Z,t), \tag{A.19}$$

where from equation (A.12), $\Pi_x(Z,t) = 0$ for firms who do not export. Let $V(Z,t)$ be the value of a firm with productivity Z at time t. The interest rate used by the firm to discount, $r(t)$, contains the death rate δ and the discount rate of the consumer.

Given the standard Bellman equation for the GBM of equation (A.18) and optimal static policies,

$$r(t)V(Z,t) = \Pi(Z,t) + \left(\mu + \frac{\upsilon^2}{2}\right) Z \frac{\partial V(Z,t)}{\partial Z} + \frac{\upsilon^2}{2} Z^2 \frac{\partial^2 V(Z,t)}{\partial Z^2} + \frac{\partial V(Z,t)}{\partial t}, \tag{A.20}$$

$$V(M(t),t) = \int_{M(t)}^{\infty} V(Z,t)\mathrm{d}\Phi(Z,t) - X(t), \tag{A.21}$$

$$\frac{\partial V(M(t),t)}{\partial Z} = 0. \tag{A.22}$$

Equation (A.20) is the bellman equation for a firm continuing to produce with it's existing technology. It receives instantaneous profits and the value of a firm where productivity may change over time. Equation (A.21) is the value matching condition, which states that the marginal adopter must be indifferent between adopting and not adopting. $M(t)$ is the endogenous productivity threshold that defines the marginal firm. Equation (A.22) is the smooth-pasting condition.[20]

[20]Using the standard relationship between free boundary and optimal stopping time problems, the firm's problem could equivalently be written as the firm choosing a stopping time at which it would upgrade. If $\upsilon > 0$ this stopping time is a random variable, otherwise it is deterministic.

A.4. Adoption Costs

In order to upgrade its technology a firm must buy some goods and hire some labor. These costs are in proportion to the population size, reflecting market access costs as in Arkolakis (2010) or Arkolakis (2015). Whether costs are in terms of goods or labor is a common issue in growth models, with many papers specifying goods costs and many specifying labor costs.[21] The growth literature often uses labor costs, since new ideas can not simply be purchased, and instead must be the result of innovators doing R&D. In the technology diffusion context, in which low productivity firms are adopting already existing technologies, an adoption cost that has some nontrivial component denominated in goods is more reasonable than in the innovation case. Since there is a paucity of empirical evidence to guide our decision in the adoption context, we model the adoption cost in a way that nests the costs being in labor exclusively, in goods exclusively, or as a mix of labor and goods. Although we solve the model for this general case, our baseline is that costs are purely labor denominated.

The amount of labor needed is parameterized by ζ, which is constant. The labor component of the adoption cost, however, increases in equilibrium in proportion to the real wage, ensuring the cost does not become increasingly small as the economy grows. The amount of goods that needs to be purchased to adopt a technology increases with the scale of the economy—otherwise the relative costs of goods would become infinitesimal in the long-run. Θ parameterizes the amount of goods required to adopt a technology, with the cost, $M(t)\,\Theta$, growing as the economy grows. Essentially, ζ controls the overall cost of technology adoption, while $\eta \in [0,1]$ controls how much of the costs are to hire labor versus buy goods. We model the mix of labor and goods as additive in order to permit a balanced growth path equilibrium.

The real cost of adopting a technology is

$$X(t) := \bar{L}\zeta\left[(1-\eta)\frac{W(t)}{P(t)} + \eta M(t)\Theta\right]. \tag{A.23}$$

A.5. Entry and Exit

There is a large pool of non-active firms that may enter the economy by paying an entry cost—equity financed by the representative consumer—to gain a draw of an initial productivity from the same distribution from which adopters draw. Since entry and adoption deliver similar gains, we model the cost of entry as a multiple of the adoption cost for incumbents, $X(t)/\chi$, where $0 < \chi < 1$. Hence, χ is the ratio of adoption to entry costs and $\chi \in (0,1)$ reflects that

[21] As discussed in Bollard, Klenow, and Li (2014), for examples see Hopenhayn (1992), Romer (1994), Foster, Haltiwanger, and Syverson (2008) and David (2014) for papers using output costs and Lucas (1978), Grossman and Helpman (1991), Melitz (2003), Klette and Kortum (2004), Luttmer (2007), and Acemoglu, Akcigit, Bloom, and Kerr (2013) for papers that use labor costs.

incumbents have a lower cost of upgrading to a better technology than entrants have to start producing a new variety from scratch.

Thus, the free entry condition that equates the cost of entry to the value of entry is

$$X(t)/\chi = \int_{M(t)}^{\infty} V(Z,t)\mathrm{d}\Phi(Z,t). \tag{A.24}$$

If a flow $\delta(t)$ of firms enter, and a flow δ exit, then the differential equation for the number of firms is $\Omega'(t) = (\delta(t) - \delta)\,\Omega(t)$. Since we study a stationary equilibrium, on a BGP Ω will be constant and determined by free entry, and $\delta(t) = \delta$ for all t.

The costs of entry will determine the number of varieties in equilibrium, and for $\delta > 0$, there is gross entry on a balanced growth path. This model of entry and exit is very different from those in Luttmer (2007) and Sampson (2015). Here, exit is exogenous, whereas a key model mechanism studied in those papers is the endogenous selection of exit induced by fixed costs of operations. We have not modeled a fixed cost to domestic production in order to isolate our distinct mechanism. We have introduced entry and exit in our model to generate an endogenous number of varieties so that we can analyze the effect of our mechanism on welfare, taking into account changes in incumbent technology adoption behavior and changes on the extensive margin in the number of varieties produced. Given the exogenous death shock, the effect of $\delta > 0$ is only to change the firm's discount rate. For the most part, the economics are qualitatively identical to the $\delta = 0$ case and there is no discontinuity in the limit as $\delta \to 0$.[22]

A.6. Market Clearing and the Resource Constraint

Total labor demand is the sum of labor used for domestic production, export production, the fixed cost of exporting, technology adoption, and entry. Equating labor supply and demand yields

$$\bar{L} = \Omega \int_{M}^{\infty} \ell_d(Z)\mathrm{d}\Phi(Z) + (N-1)\Omega \int_{\hat{Z}}^{\infty} \ell_x(Z)\mathrm{d}\Phi(Z) + (N-1)\Omega(1-\Phi(\hat{Z}))\kappa\bar{L} + \bar{L}(1-\eta)\zeta\Omega\left(S + \delta/\chi\right). \tag{A.25}$$

[22] For special cases where $\delta = 0$ and the initial Ω is large relative to that which would be achieved on a BGP from a relatively small initial Ω, the free entry condition could hold as an inequality (i.e. $X(t)/\chi > \int_{M(t)}^{\infty} V(Z,t)d\Phi(Z)$). In that case, the lack of exit would prevent Ω from decreasing so that the free entry condition held with equality. In our baseline with $\delta = 0$, we ignore this special one-sided case, as it is economically uninteresting; it is unreasonable for the number of varieties to require major decreases in a growing economy. Also, as there is no discontinuity in the solution when taking $\delta \to 0$, we will consider our baseline economy a small δ approximation.

The quantity of final goods must equal the sum of consumption and investment in technology adoption. Thus, the resource constraint is

$$\frac{Y}{P} = C + \Omega \bar{L} \eta \zeta M \Theta(S + \delta/\chi). \tag{A.26}$$

B. Deriving the Productivity Distribution Law of Motion and Flow of Adopters

This section describes the details of deriving the law of motion for the productivity distribution.

The Productivity Distribution Law of Motion. At points of continuity of $M(t)$, there exists a flow of adopters during each infinitesimal time period.[23] The Kolmogorov Forward Equation (KFE) for $Z > M(t)$, describes the evolution of the CDF. The KFE contains standard components accounting for the drift and Brownian motion of the exogenous GBM process detailed in equation (A.18). Furthermore, it includes the flow of adopters (source) times the density they draw from (redistribution CDF). Determining the flow of adopters is the fact that the adoption boundary $M(t)$ sweeps across the density from below at rate $M'(t)$. As adoption boundary acts as an absorbing barrier, and as it sweeps from below it collects $\phi(M(t), t)$ amount firms. The cdf that the flow of adopters is redistributed into is determined by two features of the environment. In the stationary equilibrium, $M(t)$ is the minimum of support of $\Phi(Z, t)$, so the adopters are redistributed across the entire support of $\phi(Z, t)$.[24] Since adopters draw directly from the productivity density, they are redistributed throughout the distribution in proportion to the density. Thus, the flow of adopters $S(t)$ multiplies the cdf, $\Phi(Z, t)$.

Since there is a constant death rate, $\delta \geq 0$, a normalized mass of $\delta\Phi(Z, t)$ exit with productivity below Z, but as new entrants of normalized flow $\delta(t)$ adopt a productivity through the same process as incumbents, they are added to the flow entering with mass below Z.[25]

[23] The evolution of $M(t)$, and hence the distribution itself, can only be discontinuous at time 0 or in response to unanticipated shocks. Since this paper analyzes balanced growth path equilibria, we omit derivation of the law of motion for the distribution with discontinuous $M(t)$.

[24] Unlike in discrete time, the distinction between drawing from the unconditional distribution or the conditional distribution of non-adopting incumbents is irrelevant. The number of adopting firms is a flow, and hence measure 0, which leads to identical conditional vs. unconditional distributions.

[25] To derive from the more common KFE written in PDFs: use the standard KFE for the pdf $\phi(Z, t)$, integrate this with respect to Z to convert into cdf $\Phi(Z, t)$, use the fundamental theorem of calculus on all terms, then the chain rule on the last term, and rearrange,

$$\frac{\partial \phi(Z,t)}{\partial t} = -\frac{\partial}{\partial Z}\left[(\mu + \frac{\upsilon^2}{2})Z\phi(Z,t)\right] + \frac{\partial}{\partial Z^2}\left[\frac{\upsilon^2}{2}Z^2\phi(Z,t)\right] + \dots \tag{B.1}$$

$$\frac{\partial \Phi(Z,t)}{\partial t} = \left(\frac{\upsilon^2}{2} - \mu\right) Z \frac{\partial \Phi(Z,t)}{\partial Z} + \frac{\upsilon^2}{2} Z^2 \frac{\partial^2 \Phi(Z,t)}{\partial Z^2} + \dots \tag{B.2}$$

$$
\begin{aligned}
\frac{\partial \Phi(Z,t)}{\partial t} = & \underbrace{\Phi(Z,t)\left(\underbrace{S(t)+\delta(t)}_{\text{Adopt or Enter}}\right)}_{\text{Distributed below } Z} - \underbrace{S(t)}_{\text{Adopt at } M(t)} - \underbrace{\delta\Phi(Z,t)}_{\text{Death}} \\
& \underbrace{-\left(\mu - \frac{\upsilon^2}{2}\right) Z \frac{\partial \Phi(Z,t)}{\partial Z}}_{\text{Deterministic Drift}} + \underbrace{\frac{\upsilon^2}{2} Z^2 \frac{\partial^2 \Phi(Z,t)}{\partial Z^2}}_{\text{Brownian Motion}}.
\end{aligned} \tag{B.3}
$$

In the baseline $\delta = \mu = \upsilon = 0$ case, this simplifies to equation (15).[26]

Normalized Productivity Distribution. Define the change of variables $z := Z/M(t)$, $g(t) := M'(t)/M(t)$, and

$$
\Phi(Z,t) =: F(Z/M(t), t). \tag{B.4}
$$

Differentiating,

$$
\phi(Z,t) = \frac{1}{M(t)} f(Z/M(t), t). \tag{B.5}
$$

This normalization generates an adoption threshold that is stationary at $z = M(t)/M(t) = 1$ for all t.

Law of Motion for the Normalized Distribution. To characterize the normalized KFE, first differentiate the cdf with respect to t, yielding

$$
\frac{\partial \Phi(Z,t)}{\partial t} = \frac{\partial F(Z/M(t),t)}{\partial t} - \frac{Z}{M(t)} \frac{M'(t)}{M(t)} \frac{\partial F(Z/M(t),t)}{\partial z}. \tag{B.6}
$$

[26] While *conditional on an optimal policy* the law of motion in equation (15) and the more general equation (B.3) are identical to that in Luttmer (2007), this is a mechanical result of any distribution evolution with resetting of agents through direct sampling of the distribution. Economically, the forces which determine the endogenous policy are completely different, as we concentrate on the choices of incumbents rather than entrants/exit. Some of the key differences are evident in the connection to search models as discussed in Section 4.

Differentiating the cdf with respect to Z yields

$$\frac{\partial \Phi(Z,t)}{\partial Z} = \frac{1}{M(t)} \frac{\partial F(Z/M(t),t)}{\partial z}, \tag{B.7}$$
$$\frac{\partial^2 \Phi(Z,t)}{\partial Z^2} = \frac{1}{M(t)^2} \frac{\partial^2 F(Z/M(t),t)}{\partial z^2}. \tag{B.8}$$

Given that $z := \frac{Z}{M(t)}$ and $g(t) := M'(t)/M(t)$, combining equations (B.3), (B.6), (B.7), and (B.8) provides the KFE in cdfs of the normalized distribution:

$$\frac{\partial F(z,t)}{\partial t} = \left(S(t) + \delta(t) - \delta\right) F(z,t) + \left(g(t) - \mu + \upsilon^2/2\right) z \frac{\partial F(z,t)}{\partial z} + \frac{\upsilon^2}{2} z^2 \frac{\partial^2 F(z,t)}{\partial z^2} - S(t). \tag{B.9}$$

The interpretation of this KFE is that while non-adopting incumbent firms are on average not moving in absolute terms, they are moving backwards at rate $g(t)$ relative to $M(t)$ (adjusted for the growth rate of the stochastic process). As the minimum of support is $z = M(t)/M(t) = 1$ for all t, a necessary condition is that $F(1,t) = 0$ for all t, and therefore $\frac{\partial F(1,t)}{\partial t} = 0$. Thus, evaluating equation (B.9) at $z = 1$ gives an expression for $S(t)$,[27]

$$S(t) = \left(g(t) - \mu + \frac{\upsilon^2}{2}\right) \frac{\partial F(1,t)}{\partial z} + \frac{\upsilon^2}{2} \frac{\partial^2 F(1,t)}{\partial z^2}. \tag{B.10}$$

This expression includes adopters caught by the boundary moving relative to their drift, as well as the flux from the GBM pushing some of them over the endogenously determined threshold. If $\mu = \upsilon = \delta = 0$, then a truncation at $M(t)$ solves equation (B.9) for any t and for any initial condition, as in Perla and Tonetti (2014).

$$\phi(Z,t) = \frac{\phi(Z,0)}{1 - \Phi(M(t),0)}. \tag{B.11}$$

The only non-degenerate stationary $F(z)$ consistent with equation (B.11) is given by equation (B.13)—the same form as that with $\upsilon > 0$.

The Stationary Normalized Productivity Distribution. From equation (B.9), the stationary KFE is

$$0 = SF(z) + \left(g - \mu + \frac{\upsilon^2}{2}\right) zF'(z) + \frac{\upsilon^2}{2} z^2 F''(z) - S. \tag{B.12}$$

[27] Equivalently, the flow of adopters can be derived as the net flow of the probability current through the adoption threshold.

subject to $F(1) = 0$ and $F(\infty) = 1$.

Moreover, for any strictly positive $\upsilon > 0$, the KFE will asymptotically generate a stationary Pareto distribution for some tail parameter from any initial condition. While many $\theta > 1$ could solve this differential equation, the particular θ tail parameter is determined by initial conditions and the evolution of $M(t)$. As in Luttmer (2007) and Gabaix (2009), the geometric random shocks leads to an endogenously determined power-law distribution.

$$F(z) = 1 - z^{-\theta}. \tag{B.13}$$

From equations B.12 and B.13, evaluating at $z = 1$ for a given g and θ,

$$S = \theta \left(g - \mu - \theta \frac{\upsilon^2}{2} \right) \tag{B.14}$$

Therefore, given an equilibrium g and θ, the CDF in equation (B.13) and S from equation (B.14) characterize the stationary distribution. The relationship between g and θ is determined by the firms' decisions given S and $F(z)$. It is independent of δ on a BGP since the exit rate is constant and uniform across firms (in contrast to Luttmer (2007), where selection into exit is generated by fixed costs of operations and is not independent of firm productivity).

The Stationary Distribution with no GBM. For $\upsilon = 0$, the lack of random shocks means that the stationary distribution will not necessarily become a Pareto distribution from arbitrary initial conditions. However, if the initial distribution is Pareto, the normalized distribution will be constant. If the initial distribution is a power-law, then the stationary distribution is asymptotically Pareto.

Since a Pareto distribution is attained for any $\upsilon > 0$, we consider our baseline ($\upsilon = 0$, $\mu = 0$) case with an initial Pareto distribution as a small noise limit of the full model with GBM from some arbitrary initial condition. Beyond endogenously determining the tail index and changing the expected time to execute the adoption option, exogenous productivity volatility of incumbent firms has qualitatively little impact on the model. For the baseline case, from equation (B.14), the flow of adopters is $S = \theta g$.

C. Normalized Static Equilibrium Conditions

To aid in computing a balanced growth path equilibrium, in this section we transform the problem and derive normalized static equilibrium conditions.

Expectations using the Normalized Distribution. If an integral of the following form exists, for some unary function $\Psi(\cdot)$, substitute for $f(z)$ from B.5, then do a change of variables of $z = \frac{Z}{M}$ to obtain a useful transformation of the integral.

$$\int_M^\infty \Psi\left(\tfrac{Z}{M}\right)\phi(Z)\mathrm{d}Z = \int_M^\infty \Psi\left(\tfrac{Z}{M}\right) f\left(\tfrac{Z}{M}\right)\tfrac{1}{M}\mathrm{d}Z = \int_{M/M}^\infty \Psi(z)f(z)\mathrm{d}z. \tag{C.1}$$

The key to this transformation is that moving from ϕ to f introduces a $1/M$ term. Thus, abusing notation by using an expectation of the normalized variable,

$$\int_M^\infty \Psi\left(\tfrac{Z}{M}\right)\phi(Z)\mathrm{d}Z = \mathbb{E}\left[\Psi(z)\right]. \tag{C.2}$$

C.1. Normalizing the Static Equilibrium

Define the following normalized, real, per-capita values: $\hat{z} := \frac{\hat{Z}}{M}, y := \frac{Y}{\bar{L}MP}, c := \frac{C}{\bar{L}M}, q_d(Z) := \frac{Q_d(Z)}{\bar{L}M}, x := \frac{X}{\bar{L}Mw}, w := \frac{W}{MP}$ and $\pi_d(Z) := \frac{\Pi_d(Z)}{\bar{L}Mw}$. Note the normalization of profits and adoption costs is relative to real, normalized wages.

Combining the normalized variables with equations (A.8) and (A.9) provides the real prices in terms of real, normalized wages.

$$\frac{p_d(Z)}{P} = \bar{\sigma}\frac{w}{Z/M}, \tag{C.3}$$

$$\frac{p_x(Z)}{P} = \bar{\sigma}d\frac{w}{Z/M}. \tag{C.4}$$

Substituting equations (C.3) and (C.4) into equation (A.4) and dividing by $\bar{L}M$ yields normalized quantities,

$$q_d(Z) = \bar{\sigma}^{-\sigma}w^{-\sigma}y\left(\tfrac{Z}{M}\right)^\sigma, \tag{C.5}$$

$$q_x(Z) = \bar{\sigma}^{-\sigma}w^{-\sigma}d^{-\sigma}y\left(\tfrac{Z}{M}\right)^\sigma. \tag{C.6}$$

Divide equation (A.10) by $\bar{L}$, then substitute from equations (A.4) and (C.3). Finally, divide the top and bottom by M to obtain normalized demand for production labor

$$\ell_d(Z)/\bar{L} = \bar{\sigma}^{-\sigma}w^{-\sigma}y\left(\tfrac{Z}{M}\right)^{\sigma-1}, \tag{C.7}$$

$$\ell_x(Z)/\bar{L} = \bar{\sigma}^{-\sigma}w^{-\sigma}yd^{1-\sigma}\left(\tfrac{Z}{M}\right)^{\sigma-1}. \tag{C.8}$$

Divide equation (A.5) by $P^{1-\sigma}$ and then substitute from equation (C.3) for $p_d(Z)/P$ to obtain

$$1 = \Omega\bar{\sigma}^{1-\sigma}w^{1-\sigma}\left[\int_M^\infty \left(\tfrac{Z}{M}\right)^{\sigma-1}\mathrm{d}\Phi(Z) + (N-1)\int_{\hat{Z}}^\infty d^{1-\sigma}\left(\tfrac{Z}{M}\right)^{\sigma-1}\mathrm{d}\Phi(Z)\right]. \tag{C.9}$$

Simplify equation (C.9) by defining $\bar{z}$, a measure of effective aggregate productivity. Then use equation (C.2) to give normalized real wages in terms of parameters, $\hat{z}$, and the productivity distribution

$$\bar{z} := \left[\Omega\left(\mathbb{E}\left[z^{\sigma-1}\right] + (N-1)(1-F(\hat{z}))d^{1-\sigma}\mathbb{E}\left[z^{\sigma-1}|z>\hat{z}\right]\right)\right]^{\frac{1}{\sigma-1}}, \tag{C.10}$$

$$w^{\sigma-1} = \bar{\sigma}^{1-\sigma}\bar{z}^{\sigma-1}, \tag{C.11}$$

$$w = \tfrac{1}{\bar{\sigma}}\bar{z}. \tag{C.12}$$

Note that if $d=1$ and $\hat{z}=1$, then $w = \frac{1}{\bar{\sigma}}\left(\Omega N \mathbb{E}\left[z^{1-\sigma}\right]\right)^{1/(\sigma-1)}$. Divide equations (A.11) and (A.12) by $\bar{L}Mw$ and substitute with equation (C.11) to obtain normalized profits,

$$\pi_d(Z) = \tfrac{1}{\sigma}\left(\tfrac{p(Z)}{P}\right)^{1-\sigma}\tfrac{y}{w} = \tfrac{1}{\sigma\bar{z}^{\sigma-1}}\tfrac{y}{w}\left(\tfrac{Z}{M}\right)^{\sigma-1}, \tag{C.13}$$

$$\pi_x(Z) = \tfrac{1}{\sigma\bar{z}^{\sigma-1}}\tfrac{y}{w}d^{1-\sigma}\left(\tfrac{Z}{M}\right)^{\sigma-1} - \kappa. \tag{C.14}$$

Divide equations (A.15) and (A.16) by $\bar{L}Mw$ and use equations (C.13) and (C.14) to find aggregate profits from domestic production and from exporting to one country,

$$\bar{\pi}_d = \Omega\tfrac{1}{\sigma\bar{z}^{\sigma-1}}\tfrac{y}{w}\mathbb{E}\left[z^{\sigma-1}\right], \tag{C.15}$$

$$\bar{\pi}_x = \Omega\left(\tfrac{1}{\sigma\bar{z}^{\sigma-1}}\tfrac{y}{w}d^{1-\sigma}(1-F(\hat{z}))\mathbb{E}\left[z^{\sigma-1}|z>\hat{z}\right] - (1-F(\hat{z}))\kappa\right). \tag{C.16}$$

Divide equation (A.25) by $\bar{L}$, and aggregate the total labor demand from equations (C.7) and (C.8) to obtain normalized aggregate labor demand

$$\begin{aligned} 1 &= \Omega\bar{\sigma}^{-\sigma}w^{-\sigma}y\left(\mathbb{E}\left[z^{\sigma-1}\right] + (N-1)(1-F(\hat{z}))d^{1-\sigma}\mathbb{E}\left[z^{\sigma-1}|z>\hat{z}\right]\right) \\ &\quad + \Omega(N-1)(1-F(\hat{z}))\kappa + \Omega(1-\eta)\zeta S & \text{(C.17)} \\ &= \bar{\sigma}^{-\sigma}w^{-\sigma}y\bar{z}^{\sigma-1} + \Omega\left((N-1)(1-F(\hat{z}))\kappa + (1-\eta)\zeta\left(S+\delta/\chi\right)\right). & \text{(C.18)} \end{aligned}$$

Define $\tilde{L}$ as a normalized quantity of labor used outside of variable production. Multiply equation (C.18) by w, and use equation (C.11) to show that

$$\tilde{L} := \Omega\left[(N-1)(1-F(\hat{z}))\kappa + (1-\eta)\zeta\left(S+\delta/\chi\right)\right], \tag{C.19}$$

$$w = \tfrac{1}{\bar{\sigma}}y + \tilde{L}w, \tag{C.20}$$

$$1 = \tfrac{1}{\bar{\sigma}}\tfrac{y}{w} + \tilde{L}. \tag{C.21}$$

Reorganize to find real output as a function of the productivity distribution and labor supply (net of labor used for the fixed costs of exporting and adopting technology)

$$\frac{y}{w} = \bar{\sigma}\left(1 - \tilde{L}\right), \tag{C.22}$$

$$y = \left(1 - \tilde{L}\right)\bar{z}. \tag{C.23}$$

This equation lends interpretation to $\bar{z}$ as being related to the aggregate productivity. Substituting equation (C.22) into equations (C.13) and (C.14) to obtain a useful formulation of firm profits

$$\pi_d(Z) = \frac{1-\tilde{L}}{(\sigma-1)\bar{z}^{\sigma-1}}\left(\frac{Z}{M}\right)^{\sigma-1}, \tag{C.24}$$

$$\pi_x(Z) = \frac{1-\tilde{L}}{(\sigma-1)\bar{z}^{\sigma-1}}d^{1-\sigma}\left(\frac{Z}{M}\right)^{\sigma-1} - \kappa. \tag{C.25}$$

Define the common profit multiplier $\bar{\pi}_{\min}$ as

$$\bar{\pi}_{\min} := \frac{1-\tilde{L}}{(\sigma-1)\bar{z}^{\sigma-1}} = \frac{1-\tilde{L}}{(\sigma-1)(\bar{\sigma}w)^{\sigma-1}} = \frac{(\bar{\sigma}w)^{1-\sigma}y}{\sigma w}, \tag{C.26}$$

$$\pi_d(Z) = \bar{\pi}_{\min}\left(\frac{Z}{M}\right)^{\sigma-1}, \tag{C.27}$$

$$\pi_x(Z) = \bar{\pi}_{\min}d^{1-\sigma}\left(\frac{Z}{M}\right)^{\sigma-1} - \kappa. \tag{C.28}$$

Use equation (C.28) set to zero to solve for $\hat{z}$. This is an implicit equation as $\bar{\pi}_{\min}$ is a function of $\hat{z}$ through $\bar{z}$

$$\hat{z} = d\left(\frac{\kappa}{\bar{\pi}_{\min}}\right)^{\frac{1}{\sigma-1}}. \tag{C.29}$$

Substitute equations (C.27) and (C.28) into equations (C.15) and (C.16) to obtain a useful formulation for aggregate profits

$$\bar{\pi}_d = \Omega\bar{\pi}_{\min}\mathbb{E}\left[z^{\sigma-1}\right], \tag{C.30}$$

$$\bar{\pi}_x = \Omega\left((1 - F(\hat{z}))\left(\bar{\pi}_{\min}d^{1-\sigma}\mathbb{E}\left[z^{\sigma-1}|z > \hat{z}\right] - \kappa\right)\right). \tag{C.31}$$

Combine to calculate aggregate total profits

$$\begin{aligned}\bar{\pi}_d + (N-1)\bar{\pi}_x = {} & \Omega\bar{\pi}_{\min}\left[\mathbb{E}\left[z^{\sigma-1}\right] + (N-1)(1-F(\hat{z}))d^{1-\sigma}\mathbb{E}\left[z^{\sigma-1}|z > \hat{z}\right]\right] \\ & - \Omega(N-1)(1-F(\hat{z}))\kappa. \end{aligned} \tag{C.32}$$

Rewriting aggregate total profits using the definition of $\bar{z}$ yields

$$\bar{\pi}_{\text{agg}} := \bar{\pi}_{\min}\bar{z}^{\sigma-1} - \Omega(N-1)(1-F(\hat{z}))\kappa. \tag{C.33}$$

Note that in a closed economy, $\bar{z} = \left(\Omega\mathbb{E}\left[z^{\sigma-1}\right]\right)^{1/(\sigma-1)}$ and therefore aggregate profits relative to wage are a markup dependent fraction of normalized output relative to wage $\bar{\pi}_d = \frac{1-\tilde{L}}{\sigma-1}$. Take the resource constraint in equation (A.26) and divide by $M\bar{L}w$ and then use equation (C.22) to get an equation for normalized, per-capita consumption

$$\frac{c}{w} = \frac{y}{w} - \Omega\eta\zeta\Theta\left(S+\delta/\chi\right)/w = \bar{\sigma}\left(1-\tilde{L}\right) - \Omega\eta\zeta\frac{\Theta}{w}\left(S+\delta/\chi\right), \tag{C.34}$$

$$c = \left(1-\tilde{L}\right)\bar{z} - \eta\zeta\Omega\Theta\left(S+\delta/\chi\right). \tag{C.35}$$

Normalize the cost in equation (A.23) by dividing by $\bar{L}PMw$. This is implicitly a function of $\hat{z}$ through w

$$x = \zeta\left(1-\eta+\eta\Theta/w\right). \tag{C.36}$$

Normalize the trade share in equation (A.17) by substituting from equations (C.4) and (C.11)

$$\lambda = (1-F(\hat{z}))d^{1-\sigma}\frac{\Omega\mathbb{E}\left[z^{\sigma-1}|z>\hat{z}\right]}{\bar{z}^{\sigma-1}}. \tag{C.37}$$

Starting from C.33, use C.26, and C.19, to derive that

$$\bar{\pi}_{\text{agg}} = \frac{1}{\sigma-1}\left(1-\Omega\left[(1-\eta)\zeta\left(S+\delta/\chi\right)-\sigma(N-1)(1-F(\hat{z}))\kappa\right]\right). \tag{C.38}$$

Stationary Trade Shares and Profits. Using the stationary distribution in equation (B.13), calculate average profits from C.33,

$$\frac{\bar{\pi}_{agg}}{\Omega} = \frac{\bar{\pi}_{\min}\theta}{1+\theta-\sigma} + \frac{(\sigma-1)(N-1)\kappa\hat{z}^{-\theta}}{1+\theta-\sigma}. \tag{C.39}$$

Note that the minimum profits are at $z=1$ and equal to $\bar{\pi}_{\min}$ as long as $\hat{z}>1$. Using this to define the profit spread between the average and worst firm in the economy,

$$\frac{\bar{\pi}_{agg}}{\Omega} - \bar{\pi}_{\min} = \underbrace{\frac{\bar{\pi}_{\min}\theta}{1+\theta-\sigma} + \frac{(\sigma-1)(N-1)\kappa\hat{z}^{-\theta}}{1+\theta-\sigma} - \bar{\pi}_{\min}}_{\text{ProfitSpread}} = \frac{(\sigma-1)\bar{\pi}_{\min}}{1+\theta-\sigma} + \frac{(\sigma-1)(N-1)\kappa\hat{z}^{-\theta}}{1+\theta-\sigma}. \tag{C.40}$$

Define the ratio of mean to minimum profits as $\bar{\pi}_{\text{rat}} := \frac{\bar{\pi}/\Omega}{\bar{\pi}_{\min}}$. From C.40 and C.29 find that

$$\bar{\pi}_{\text{rat}} = \frac{\theta}{1+\theta-\sigma} + (N-1)d^{1-\sigma}\frac{(\sigma-1)\hat{z}^{\sigma-1-\theta}}{1+\theta-\sigma}. \tag{C.41}$$

Take equations (C.37) and (C.10) to find

$$\bar{z}^{\sigma-1} = \Omega\mathbb{E}\left[z^{\sigma-1}\right] + (N-1)\lambda\bar{z}^{\sigma-1}. \tag{C.42}$$

Solving gives an expression for a function of aggregate productivity in terms of underlying productivity and trade shares. Defining the home trade share as $\lambda_{ii} := 1-(N-1)\lambda$,

$$\bar{z}^{\sigma-1} = \Omega\frac{\mathbb{E}\left[z^{\sigma-1}\right]}{1-(N-1)\lambda} = \Omega\frac{\mathbb{E}\left[z^{\sigma-1}\right]}{\lambda_{ii}}. \tag{C.43}$$

From equations (C.12) and (C.43),

$$w = \frac{1}{\bar{\sigma}}\Omega^{\frac{1}{1-\sigma}}\mathbb{E}\left[z^{\sigma-1}\right]^{\frac{1}{\sigma-1}}\lambda_{ii}^{\frac{1}{1-\sigma}}. \tag{C.44}$$

This relates the real normalized wage to the aggregate productivity, the home trade share, and the number of varieties. Given that $\sigma > 1$, this expression implies that the larger the share of goods purchased at home, the lower the real wage is.

From C.37 and C.43 ,

$$\lambda = (1-F(\hat{z}))d^{1-\sigma}\frac{\mathbb{E}\left[z^{\sigma-1}|z>\hat{z}\right]\lambda_{ii}}{\mathbb{E}\left[z^{\sigma-1}\right]}. \tag{C.45}$$

Using the stationary distribution,

$$\lambda = \hat{z}^{-\theta}d^{1-\sigma}\frac{\hat{z}^{\sigma-1}\frac{\theta}{\theta-(\sigma-1)}\lambda_{ii}}{\frac{\theta}{\theta-(\sigma-1)}} = \hat{z}^{-\theta}d^{1-\sigma}\hat{z}^{\sigma-1}\lambda_{ii}. \tag{C.46}$$

Using the definition of the home trade share,

$$\lambda_{ii} = \frac{1}{1+(N-1)\hat{z}^{\sigma-1-\theta}d^{1-\sigma}}. \tag{C.47}$$

Furthermore, multiplying the numerator and denominator by $\bar{\pi}_{\min}$ and using equation (C.29),

$$\lambda_{ii} = \frac{\bar{\pi}_{\min}}{\bar{\pi}_{\min}+(N-1)\hat{z}^{-\theta}\kappa}. \tag{C.48}$$

Which gives an alternative expression for $\bar{\pi}_{\min}$ when $\kappa > 0$,

$$\bar{\pi}_{\min} = \frac{(N-1)\hat{z}^{-\theta}\kappa}{1-\lambda_{ii}}. \tag{C.49}$$

D. Normalized and Stationary Dynamic Equilibrium Conditions

This section derives normalized stationary dynamic balanced growth path equilibrium conditions.

D.1. Utility and Welfare on a BGP

Using the substitution $C(t) = c\bar{L}M(t)$ shows time 0 welfare $\bar{U}$ as a function of c and g is

$$\bar{U}(c,g) = \tfrac{1}{1-\gamma}\tfrac{(c\bar{L}M(0))^{1-\gamma}}{\rho+(\gamma-1)g}. \tag{D.1}$$

With $\log$ utility

$$\bar{U}(c,g) = \frac{\rho\log(c\bar{L}M(0)) + g}{\rho^2}. \tag{D.2}$$

Using the standard IES of the consumer, adjusted for stochastic death of the firm, the interest rate on a BGP is,

$$r = \rho + \gamma g + \delta. \tag{D.3}$$

With $\log$ utility

$$r = \rho + g + \delta. \tag{D.4}$$

We restrict parameters such that $g(1-\gamma) < \rho$ in equilibrium to ensure finite utility. In the $\log$ utility case, this is simply $\rho > 0$.

D.2. Normalization of the Firm's Dynamic Problem

We proceed to derive the normalized continuation value function, value matching condition, and smooth pasting condition originally specified in equations (A.20)–(A.22). Define the normalized real value of the firm relative to normalized wages as

$$v(z,t) := \tfrac{V(Z,t)}{\bar{L}M(t)w(t)}. \tag{D.5}$$

Rearranging

$$V(Z,t) = \bar{L}w(t)M(t)v(Z/M(t),t). \tag{D.6}$$

First differentiate the continuation value $V(Z,t)$ with respect to t in equation (D.6) and divide by $w(t)M(t)\bar{L}$, using the chain and product rule. This gives,

$$\frac{1}{w(t)M(t)\bar{L}}\frac{\partial V(Z,t)}{\partial t} = \frac{M'(t)}{M(t)}v(z,t) - \frac{M'(t)}{M(t)}\frac{Z}{M(t)}\frac{\partial v(z,t)}{\partial z} + \frac{M(t)}{M(t)}\frac{\partial v(z,t)}{\partial t} + \frac{w'(t)}{w(t)}v(z,t). \tag{D.7}$$

Defining the growth rate of $g(t) := M'(t)/M(t)$ and $g_w(t) := w'(t)/w(t)$. Substitute these into equation (D.7), cancel out $M(t)$, and group $z = Z/M(t)$ to give

$$\frac{1}{w(t)M(t)\bar{L}}\frac{\partial V(Z,t)}{\partial t} = (g(t)+g_w(t))v(z,t) - g(t)z\frac{\partial v(z,t)}{\partial z} + \frac{\partial v(z,t)}{\partial t}. \tag{D.8}$$

Differentiating equation (D.6) with respect to Z yields

$$\frac{\partial V(Z,t)}{\partial Z} = \frac{\bar{L}M(t)w(t)}{M(t)}\frac{\partial v(Z/M(t),t)}{\partial z} = \bar{L}w(t)\frac{\partial v(z,t)}{\partial z}. \tag{D.9}$$

Similarly,

$$\frac{\partial^2 V(Z,t)}{\partial Z^2} = \frac{\bar{L}w(t)}{M(t)}\frac{\partial^2 v(z,t)}{\partial z^2}. \tag{D.10}$$

Define the normalized profits from equation (A.19) as

$$\pi(z,t) := \frac{\Pi(zM(t),t)}{w(t)M(t)\bar{L}}. \tag{D.11}$$

Divide equation (A.20) by $M(t)w(t)\bar{L}$, then substitute for $\frac{\partial V(Z,t)}{\partial t}$, $\frac{\partial V(Z,t)}{\partial Z}$, and $\frac{\partial^2 V(Z,t)}{\partial Z^2}$ from eqs. (D.8), (D.9), and (D.10) in to (A.20). Finally, group the normalized profits using equation (D.11):

$$(r(t)-g(t)-g_w(t))v(z,t) = \pi(z,t) + \left(\mu + \frac{\upsilon^2}{2} - g(t)\right)z\frac{\partial v(z,t)}{\partial z} + \frac{\upsilon^2}{2}z^2\frac{\partial^2 v(z,t)}{\partial z^2} + \frac{\partial v(z,t)}{\partial t}. \tag{D.12}$$

Equation (D.12) is the normalized version of the value function of the firm in the continuation region. The stationary version of this equation is,

$$(r-g)v(z) = \pi(z) + \left(\mu + \frac{\upsilon^2}{2} - g\right) z v'(z) + \frac{\upsilon^2}{2} z^2 v''(z). \tag{D.13}$$

To derive the normalized smooth pasting condition, use equation (A.22) to show that equation (D.9) evaluated at $Z = M(t)$ equals 0, delivering

$$\frac{\partial v(1,t)}{\partial z} = 0. \tag{D.14}$$

To arrive at the normalized value matching condition, divide equation (A.21) by $M(t)w(t)\bar{L}$ to obtain

$$\frac{V(M(t),t)}{M(t)w(t)\bar{L}} = \int_{M(t)}^{\infty} \frac{V(Z,t)}{M(t)w(t)\bar{L}} \phi(Z,t)\mathrm{d}Z - \frac{X(t)}{M(t)w(t)\bar{L}} \tag{D.15}$$

Substituting using equation (D.5) and the definition of $x(t)$ yields

$$v(M(t)/M(t),t) = \int_{M}^{\infty} v(Z/M,t)\phi(Z,t)\mathrm{d}Z - x(t). \tag{D.16}$$

Finally, normalize the integral, realizing it is of the form discussed in equation (C.2), to obtain the normalized value matching condition:

$$v(1,t) = \int_{1}^{\infty} v(z,t) f(z,t)\mathrm{d}z - x(t). \tag{D.17}$$

D.3. Normalization of the Free Entry Condition

Normalizing the free entry condition given in equation (A.24), following similar steps that delivered the normalized value matching condition in equation (D.17), gives

$$x(t)/\chi = \int_{1}^{\infty} v(z,t) f(z,t)\mathrm{d}z. \tag{D.18}$$

Relating this to the value-matching condition of the adopting firm given in equation (D.17) provides a simple formulation of the stationary free entry condition that is useful in determining Ω and g:

$$v(1) = x\frac{1-\chi}{\chi}. \tag{D.19}$$

E. Solving for the Continuation Value Function

Although our baseline model does not feature exogenous productivity shocks, in this section we solve for the value function of a more general model that has GBM with $\upsilon \geq 0$. Our baseline case of $\upsilon = 0$ is nested in this formulation. The differential equation for the value function is solved using the method of undetermined coefficients. The goal of this section is to solve for the value function as a function of parameters, g, Ω, and $\hat{z}$ (sometimes implicitly through $\bar{\pi}_{min}$).

Selection into Exporting. If $\kappa > 0$, generically some firms will choose to be exporters and some firms will only sell domestically. The value function will have a region of productivities representing the value of firms that only sell domestically and a region representing firms that also export. That is,

$$v(z) = \begin{cases} v_d(z) & \text{if } z \leq \hat{z} \\ v_x(z) & \text{if } z \geq \hat{z}. \end{cases}$$

We guess the value function is of the following form, with undetermined constants a, ν, and b,[28]

$$v_d(z) = a\bar{\pi}_{\min}\left(z^{\sigma-1} + \frac{\sigma-1}{\nu}z^{-\nu}\right), \tag{E.1}$$

$$v_x(z) = a\bar{\pi}_{\min}\left((1+(N-1)d^{1-\sigma})z^{\sigma-1} + \frac{\sigma-1}{\nu}z^{-\nu} + (N-1)\frac{1}{a(r-g)}\left(bz^{-\nu} - \frac{\kappa}{\bar{\pi}_{\min}}\right)\right). \tag{E.2}$$

The value of a firm can be decomposed into the value of operating with its current productivity forever and the option value of adopting a better technology. The constant a is a discounting term on the value of earning the profits from producing with productivity z in perpetuity. The constant ν reflects the rate at which the option value of technology adoption goes to zero as productivity increases. The constant b is an adjustment to the perpetuity profits that reflects a firm with productivity z will eventually switch from exporting to being a domestic producer if z is constant in a growing economy.

By construction, the form of these guesses ensures that value matching and smooth pasting are satisfied, both at the adoption threshold ($z = 1$) and the exporter threshold ($z = \hat{z}$). To solve for a and ν, substitute equation (E.1) into the continuation value function in equation (D.13) using $\pi_d(z)$ from equation (C.27). This generates an ODE and, grouping terms, the method of

[28]This guess is also applying a standard transversality condition to eliminate an explosive root.

undetermined coefficients provides a system of 2 equations in the 2 unknowns.[29]

Solving the system gives,

$$\nu = \frac{\mu - g}{\upsilon^2} + \sqrt{\left(\frac{g-\mu}{\upsilon^2}\right)^2 + \frac{r-g}{\upsilon^2/2}}, \tag{E.3}$$

$$a = \frac{1}{r - g - (\sigma - 1)(\mu - g + (\sigma - 1)\upsilon^2/2)}. \tag{E.4}$$

To solve for b, plug equation (E.2) into the continuation value function in equation (D.13) using $\pi_x(z)$ from equation (C.28). This generates an ODE and, grouping terms, the method of undetermined coefficients provides a system of 3 equations in the 3 unknowns. By construction, the a and ν terms match those previously found, giving a consistent solution for b:

$$b = (1 - a(r-g))\, d^{1-\sigma} \hat{z}^{\nu+\sigma-1}. \tag{E.5}$$

Note, the main effect of the GBM is to modify the ν constant to reflect changes in the expected execution time of the option value of technology diffusion, and hence the exponent for discounting.

As will be useful in solving for g, evaluating at the adoption threshold yields,

$$v(1) = a\bar{\pi}_{\min}\left(1 + \frac{\sigma - 1}{\nu}\right). \tag{E.6}$$

For the baseline case of $\mu = \upsilon = 0$,

$$a = \frac{1}{r + (\sigma - 2)g}, \tag{E.7}$$

$$\nu = \frac{r}{g} - 1, \tag{E.8}$$

$$b = \frac{\sigma - 1}{\nu + \sigma - 1} d^{1-\sigma} \hat{z}^{\nu+\sigma-1}. \tag{E.9}$$

[29]Instead of the method of undetermined coefficients, a direct solution approach would be to solve the continuation value function ODEs in the domestic sales and exporter regions, using the smooth pasting condition as the boundary value.

F. Computing the BGP Equilibrium when All Firms Export

In the case of $\kappa = 0$ all firms export (given d s.t. the economy is not in autarky), and the value function has only one region. We guess that the value function will take the following form,

$$v(z) = a\bar{\pi}_{\min}\left(1+(N-1)d^{1-\sigma}\right)\left(z^{\sigma-1} + \frac{\sigma-1}{\nu}z^{-\nu}\right). \tag{F.1}$$

Substituting equation (F.1) into the continuation value function in equation (D.13) with profits from equation (C.28), the ν and a are identical to those from equations (E.3) and (E.4). Evaluating at the threshold,

$$v(1) = a\left(1+(N-1)d^{1-\sigma}\right)\bar{\pi}_{\min}\left(1+\frac{\sigma-1}{\nu}\right). \tag{F.2}$$

Solving for the Growth Rate and Mass of Varieties when All Firms Export. Using the free entry condition from equation (D.19) with equation (F.2) to find,

$$\frac{x}{\bar{\pi}_{\min}} = a\left(1+(N-1)d^{1-\sigma}\right)\frac{\chi}{1-\chi}\frac{\sigma+\nu-1}{\nu}. \tag{F.3}$$

Substitute equations (F.1) and (F.2) into the value matching condition of equation (D.17), and divide by $a\bar{\pi}_{\min}(1+(N-1)d^{1-\sigma})$

$$1+\frac{\sigma-1}{\nu} = \frac{\theta(\nu+\sigma-1)(\theta+\nu-\sigma+1)}{\nu(\theta+\nu)(\theta-\sigma+1)} - \frac{x}{\bar{\pi}_{\min}a\left(1+(N-1)d^{1-\sigma}\right)}. \tag{F.4}$$

Combine equations (F.3) and (F.4), and solve for ν. For any cost function x and minimum profits $\bar{\pi}_{\min}$,

$$\nu = \frac{\chi\theta(\theta+1-\sigma)}{\sigma-1-\theta\chi}. \tag{F.5}$$

The aggregate growth rate is found by equating equations (E.3) and (F.5) to find

$$g = \underbrace{\mu}_{\text{Drift}} + \underbrace{\frac{(r-\mu)((\sigma-1)/\chi-\theta)}{\theta^2-\theta\sigma+(\sigma-1)/\chi}}_{\text{Deterministic}} + \frac{\upsilon^2}{2}\underbrace{\frac{\theta^2(\theta+1-\sigma)^2}{(\theta^2-\theta\sigma+(\sigma-1)/\chi)(\theta-(\sigma-1)/\chi)}}_{\text{Stochastic}}. \tag{F.6}$$

In the baseline case of $\upsilon = \mu = 0$

$$g = \frac{(\rho+\delta)(\sigma-1-\chi\theta)}{\theta\chi(\gamma+\theta-\sigma)-(\gamma-1)(\sigma-1)}, \tag{F.7}$$

and,

$$\frac{x}{\bar{\pi}_{\min}} = \left(1 + (N-1)d^{1-\sigma}\right)\frac{\chi}{1-\chi}\frac{1}{r-g}. \tag{F.8}$$

The growth rate is independent of the trade costs, the population, and the number of countries. The intuition–as discussed in the body of the paper–is that the growth rate is driven by the ratio of the minimum to the mean profits, which are proportional and independent of the scale or integration of economies in the absence of any export selection. The constant death rate only enters to increase the discount rate.

Note in the baseline case of $\upsilon = \mu = \delta = 0$ and $\log$ utility

$$g = \frac{\rho(\sigma - 1 - \chi\theta)}{\theta^2\chi}\bar{\pi}^k_{\text{rat}}, \tag{F.9}$$

where (from eqs. C.33 and C.10 with $\kappa = 0$ and $\hat{z} = 1$) the ratio of average profits to minimum profits is

$$\bar{\pi}^k_{\text{rat}} = \frac{\bar{\pi}^k_{\text{agg}}/\Omega}{\bar{\pi}^k_{\min}} = \frac{(1+(N-1)d^{1-\sigma})\bar{\pi}_{\min}E[z^{\sigma-1}]}{(1+(N-1)d^{1-\sigma})\bar{\pi}_{\min}} = E[z^{\sigma-1}] = \frac{\theta}{1+\theta-\sigma}. \tag{F.10}$$

The Mass of Varieties Ω. Here we solve for the mass of varieties Ω in the baseline case of $\upsilon = \mu = \delta = 0$.

Substitute into the free entry condition equation (F.8) using the definition of $\bar{\pi}_{\min}$ in terms of $\bar{z}$ and $\tilde{L}$ from equation (C.26), the definition of $\bar{z}$ from equation (C.10), and the definition of $\tilde{L}$ from equation (C.19), to obtain the implicit equation. In the baseline case where $\eta = 0$, an explicit solution is[30]

$$\Omega = \frac{\chi((\gamma-1)(\sigma-1) - \theta\chi(\gamma+\theta-\sigma))}{\zeta\left(\theta\chi(-\gamma\delta - \sigma(\theta(\delta+\rho)+\rho) + \delta + \theta\rho + \rho) + (\gamma-1)\delta(\sigma-1) + \theta^2\sigma\chi^2(\delta+\rho)\right)}. \tag{F.13}$$

Note that the only place that the adoption cost, ζ, has come into the system of Ω and g is in the denominator of F.13. For this reason, the ζ parameter (along with $\bar{L}$) determines the scale of the

[30]The solution for the simplest case of $\gamma = 1$ and $\delta = 0$ is,

$$g = \frac{\rho(\sigma - 1 - \theta\chi)}{\theta\chi(1+\theta-\sigma)} \tag{F.11}$$

$$\Omega = \frac{\chi(1+\theta-\sigma)}{\zeta\rho\left((1+\theta)(\sigma-1) - \sigma\theta\chi\right)} \tag{F.12}$$

economy.

It can be shown that in the Krugman model for all cases with $\eta = 0$, the number of domestic varieties is independent of trade costs d. Thus, both Ω and g are independent of d if $\eta = 0$. This implies through equations (C.19) and (B.14) that the amount of labor dedicated to technology adoption, $\tilde{L}$, is also independent of d if $\eta = 0$.

Through C.35, since $\tilde{L}$, Ω, and g are independent of d when $\eta = 0$, in response to a decrease in trade costs d, c increases only due to the $(1 + (N-1)d^{1-\sigma})^{1/(\sigma-1)}$ term in $\bar{z}$.

The key relationship can be summarized by the following elasticities.

$$\frac{\partial \log \bar{\pi}^k_{\text{rat}}(d)}{\partial \log d} = 0. \tag{F.14}$$

Using equation (C.47) with $\hat{z} = 1$ shows

$$\frac{\partial \log \lambda_{ii}(d)}{\partial \log d} = (\sigma - 1)\left(1 + \frac{d^{\sigma-1}}{N-1}\right)^{-1} = (\sigma - 1)(1 - \lambda_{ii}) > 0. \tag{F.15}$$

Furthermore, when $\eta = 0$,

$$\frac{\partial \log(1 - \tilde{L}(d))}{\partial \log d} = \frac{\partial \log \Omega(d)}{\partial \log d} = \frac{\partial \log g(d)}{\partial \log d} = 0. \tag{F.16}$$

For the case with $\eta = 0$, the ratio of c for different trade costs d_1 and d_2 that both feature positive trade is

$$\frac{\bar{c}_{d_1}}{\bar{c}_{d_2}} = \frac{\bar{z}_{z_{d_1}}}{\bar{z}_{d_2}} = \left(\frac{1 + (N-1)d_1^{1-\sigma}}{1 + (N-1)d_2^{1-\sigma}}\right)^{\frac{1}{\sigma-1}}. \tag{F.17}$$

Using the normalized welfare function in equation (D.1), since g is independent of d in the $\kappa = 0$ case, the ratio of welfare for different trade costs d_1 and d_2 that both feature positive trade (for $\gamma > 0$) is,

$$\frac{\bar{U}_{d_1}}{\bar{U}_{d_2}} = \left(\frac{1 + (N-1)d_1^{1-\sigma}}{1 + (N-1)d_2^{1-\sigma}}\right)^{\frac{1-\gamma}{\sigma-1}}. \tag{F.18}$$

Comparing welfare between free trade ($d = 1$) and autarky (sufficiently high d s.t. there are no exporters) gives (for $\gamma \neq 1$)

$$\frac{\bar{U}_{free}}{\bar{U}_{autarky}} = N^{\frac{1-\gamma}{\sigma-1}}. \tag{F.19}$$

G. Computing the BGP Equilibrium with Selection into Exporting ($\kappa > 0$)

We solve for g and Ω by reducing the equilibrium conditions to a system of two equations in these two unknowns. First, combining the free entry condition from equation (D.19) with $v(1)$ from equation (E.6) yields

$$\frac{x}{\bar{\pi}_{\min}} = a\frac{\chi}{1-\chi}\frac{\sigma+\nu-1}{\nu}. \tag{G.1}$$

The second equation is found by evaluating the value matching condition of equation (D.17) by substituting in the domestic and exporter value functions in equations E.1 and E.2, the export threshold $\hat{z}$ in equation (C.29), and the value at the adoption threshold $v(1)$ in equation (E.6) and dividing by $a\bar{\pi}_{\min}$. That is, evaluate

$$\frac{v(1)}{a\bar{\pi}_{\min}} = \frac{\int_1^\infty v(z,t)f(z,t)\mathrm{d}z}{a\bar{\pi}_{\min}} - \frac{x}{a\bar{\pi}_{\min}} \tag{G.2}$$

to obtain

$$1+\frac{\sigma-1}{\nu} = \frac{\frac{\nu(n-1)(\theta-\sigma+1)\left(d^{1-\sigma}(\theta+\nu)\hat{z}^{-\theta+\sigma-1}-b\theta\hat{z}^{-\theta-\nu}\right)}{a(g-r)}+\theta\left(\nu(n-1)d^{1-\sigma}(\theta+\nu)\hat{z}^{-\theta+\sigma-1}+(\nu+\sigma-1)(\theta+\nu-\sigma+1)\right)}{\nu(\theta+\nu)(\theta-\sigma+1)} - \frac{\chi}{1-\chi}\frac{\sigma+\nu-1}{\nu}. \tag{G.3}$$

As detailed in Section H, use the definition of $\bar{\pi}_{\min}$ and substitute for $x, \nu, a, b, \hat{z}$ and r into equations (G.1) and (G.3) to find a system of 2 equations in Ω and g. Note that the adoption cost x does not appear in equation (G.3), which is why g is independent of the specification of the cost of adoption. Equation (G.1) does explicitly depend on x, which is why the number of varieties is a function of the adoption cost, and ultimately why welfare is also a function of x.

G.1. Case with $\upsilon = \mu = \delta = 0$

As the GBM does not qualitatively impact the solution, we concentrate our analytical theory on the simple baseline case. The cost of adoption is a function of minimum profits, parameters, and $r - g$:

$$x = \bar{\pi}_{\min}\frac{\chi}{1-\chi}\frac{1}{r-g}. \tag{G.4}$$

Evaluating the general equation (G.3) with the substitutions for $x, \nu, a, b, \hat{z}$ and r that correspond to the baseline case of $\upsilon = \mu = \delta = 0$ yields a unique g that satisfies the value function and the value matching equations, given by the implicit equation,

$$g = \frac{\left(\sigma - 1 + (N-1)\theta d^{1-\sigma}\hat{z}^{-\theta+\sigma-1}\right)\bar{\pi}_{\text{min}} + (N-1)(-\theta+\sigma-1)\hat{z}^{-\theta}\kappa}{x(\gamma+\theta-1)(\theta-\sigma+1)} - \frac{\rho}{\gamma+\theta-1}. \tag{G.5}$$

Using $\kappa/\bar{\pi}_{\text{min}} = \hat{z}^{\sigma-1}d^{1-\sigma}$ from equation (C.29), simplify to

$$g = \frac{(\sigma-1)\left[\bar{\pi}_{\text{min}} + (N-1)\kappa\hat{z}^{-\theta}\right]}{x(\gamma+\theta-1)(\theta-\sigma+1)} - \frac{\rho}{\gamma+\theta-1}. \tag{G.6}$$

Using equation (C.40), realize this relates the growth rate to the difference between average and minimum profits

$$g = \left(\frac{\bar{\pi}_{\text{agg}}/\Omega - \bar{\pi}_{\text{min}}}{x(\gamma+\theta-1)}\right) - \frac{\rho}{\gamma+\theta-1}. \tag{G.7}$$

Substitute for x using the free entry condition in equation (G.4), use the definition of the average to minimum profit ratio $\left(\bar{\pi}_{\text{rat}} := \frac{\bar{\pi}/\Omega}{\bar{\pi}_{\text{min}}}\right)$, and substitute for r using equation (D.3) to obtain

$$g = (\rho + (\gamma-1)g)\frac{1-\chi}{\chi(\gamma+\theta-1)}\left(\bar{\pi}_{\text{rat}} - 1\right) - \frac{\rho}{\gamma+\theta-1}. \tag{G.8}$$

Solving for g gives an equation for g as a function exclusively of parameters and the ratio of average to minimum profits

$$g = \frac{\rho}{\chi\theta\left((1-\chi)\bar{\pi}_{\text{rat}} - 1\right)^{-1} + 1 - \gamma}. \tag{G.9}$$

Furthermore, with log utility $\gamma = 1$ and

$$g = \frac{\rho(1-\chi)}{\chi\theta}\bar{\pi}_{\text{rat}} - \frac{\rho}{\chi\theta}. \tag{G.10}$$

The relationship between growth and trade costs. First, note that the growth rate is increasing in the profit ratio, (since $\chi \in (0,1)$):

$$\frac{\mathrm{d}g(\bar{\pi}_{\text{rat}})}{\mathrm{d}\bar{\pi}_{\text{rat}}} = \frac{\rho\theta\chi(1-\chi)}{\left((\gamma-1)(1-(1-\chi)\bar{\pi}_{\text{rat}} + \theta\chi\right)^2} > 0. \tag{G.11}$$

To determine if whether growth is increasing in d, use the chain rule

$$\frac{\mathrm{d}g(d)}{\mathrm{d}d} = \frac{\mathrm{d}g(\bar{\pi}_{\text{rat}})}{\mathrm{d}\bar{\pi}_{\text{rat}}}\frac{\mathrm{d}\bar{\pi}_{\text{rat}}(d)}{\mathrm{d}d}. \tag{G.12}$$

Given equations (G.11) and (G.12), a sufficient condition to conclude that $\frac{\mathrm{d}g(d)}{\mathrm{d}d} < 0$ is $\frac{\mathrm{d}\bar{\pi}_{\text{rat}}(d)}{\mathrm{d}d} < 0$. To show this differentiate C.41 w.r.t. d to find,

$$\frac{\mathrm{d}\bar{\pi}_{\text{rat}}(d)}{\mathrm{d}d} \propto -\left((\sigma-1)\hat{z}(d) + d(1+\theta-\sigma)\frac{\mathrm{d}\hat{z}(d)}{\mathrm{d}d}\right). \tag{G.13}$$

Since $d > 0$, $\hat{z}(d) > 0$, $\sigma > 1$, and $1+\theta-\sigma > 0$, a sufficient condition for $\frac{\mathrm{d}\bar{\pi}_{\text{rat}}(d)}{\mathrm{d}d} < 0$ is if $\frac{\mathrm{d}\hat{z}(d)}{\mathrm{d}d} > 0$. Differentiate equation (C.29) to find

$$\frac{\mathrm{d}\hat{z}(d)}{\mathrm{d}d} \propto (\sigma-1) - \frac{d}{\bar{\pi}_{\text{min}}(d)}\frac{\mathrm{d}\bar{\pi}_{\text{min}}(d)}{\mathrm{d}d}. \tag{G.14}$$

Therefore, a sufficient condition to conclude that $\frac{\mathrm{d}\hat{z}(d)}{\mathrm{d}d} > 0$ is

$$\frac{\mathrm{d}\log\bar{\pi}_{\text{min}}(d)}{\mathrm{d}d} < \frac{\sigma-1}{d}. \tag{G.15}$$

Summarizing,

$$\text{sign}\frac{\mathrm{d}g(d)}{\mathrm{d}d} = \text{sign}\frac{\mathrm{d}\bar{\pi}_{\text{rat}}}{\mathrm{d}d} = -\text{sign}\frac{\mathrm{d}\hat{z}(d)}{\mathrm{d}d}. \tag{G.16}$$

G.2. Baseline Case with $\upsilon = \mu = \delta = \eta = 0$ and $\log$ utility.

Adding the restriction that $\eta = 0$ and $\gamma = 1$ to the adoption cost equation (C.36) and the interest rate equation (D.4) delivers the key simplifications that permit solving for the BGP equilibrium in closed form:

Calculating the Growth Rate and the Mass of Varieties.

$$x = \zeta, \tag{G.17}$$

$$r - g = \rho. \tag{G.18}$$

Using equation (G.4) gives an expression for $\bar{\pi}_{\text{min}}$ in terms of model parameters

$$\bar{\pi}_{\text{min}} = \frac{(1-\chi)\zeta\rho}{\chi}. \tag{G.19}$$

Substitute equation (G.19) into equation (C.29) to find the export threshold in terms of parameters,

$$\hat{z} = d\left(\frac{1}{\zeta}\frac{\kappa\chi}{\rho(1-\chi)}\right)^{\frac{1}{\sigma-1}}. \tag{G.20}$$

Substitute $\gamma = 1$ and equations (G.17), (G.19), and (G.20) into equation (G.6) and simplify to obtain g in closed form:

$$g = \frac{\rho(1-\chi)}{\chi\theta}\frac{\sigma-1}{(\theta-\sigma+1)}\left(1+(N-1)d^{-\theta}\left(\frac{\kappa}{\zeta}\frac{\chi}{\rho(1-\chi)}\right)^{1-\frac{\theta}{\sigma-1}}\right)-\frac{\rho}{\theta}, \tag{G.21}$$

$$= \frac{\rho(1-\chi)}{\chi\theta}\frac{\left(\theta+(N-1)(\sigma-1)d^{-\theta}\left(\frac{1}{\zeta}\frac{\kappa\chi}{\rho(1-\chi)}\right)^{1-\frac{\theta}{\sigma-1}}\right)}{(\theta-\sigma+1)}-\frac{\rho}{\chi\theta}. \tag{G.22}$$

For ease of comparison to g as a function of $\bar{\pi}_{\text{rat}}$ use equation (G.8) evaluated at $\gamma = 1$ with equation (G.21) to realize

$$\bar{\pi}_{\text{rat}} - 1 = \frac{\left(1+(N-1)d^{-\theta}\left(\frac{1}{\zeta}\frac{\kappa\chi}{\rho(1-\chi)}\right)^{1-\frac{\theta}{\sigma-1}}\right)}{\left(\frac{\theta}{\sigma-1}-1\right)}, \tag{G.23}$$

or equation (G.10) with equation (G.22) to see

$$\bar{\pi}_{\text{rat}} = \frac{\left(\theta+(N-1)(\sigma-1)d^{-\theta}\left(\frac{1}{\zeta}\frac{\kappa\chi}{\rho(1-\chi)}\right)^{1-\frac{\theta}{\sigma-1}}\right)}{(\theta-\sigma+1)}. \tag{G.24}$$

Note, g is decreasing in d and κ (since $\theta > 0$, $\sigma > 1$, and $1+\theta-\sigma > 0$). Thus, from equation (G.16) or direct differentiation of equation (G.21),

$$\frac{\mathrm{d}g}{\mathrm{d}d} < 0; \quad \frac{\mathrm{d}\bar{\pi}_{\text{rat}}}{\mathrm{d}d} < 0; \quad \frac{\mathrm{d}\hat{z}}{\mathrm{d}d} > 0; \quad \frac{\mathrm{d}g}{\mathrm{d}\kappa} < 0. \tag{G.25}$$

See by comparing to equation (F.7) that the limit of g as $d \to \infty$ in equation (G.22) equals the autarky and all-export economy growth rates, so there is no discontinuity in the economy in this direction.

Note that the parameter ζ (previously interpreted as the scale in equation F.13) and κ only enter the growth rate multiplicatively. This is because the fixed costs of adoption, entry, and export in levels are proportional to the scale of the economy. Since the calibration strategy targets relative moments (i.e., proportion of exporters, relative size of exporters to domestic firms, growth rates, trade shares), κ is not separately identifiable from ζ without some moment that targets the level of the economy.

To find the number of varieties, maintain $\gamma = 1, \eta = 0$. To solve for Ω, start with the definition of $\bar{\pi}_{\min}$ from equation (C.26):

$$\bar{\pi}_{\min} = \tfrac{1-\tilde{L}}{(\sigma-1)\bar{z}^{\sigma-1}}. \tag{G.26}$$

For $\bar{\pi}_{\min}$, substitute from equation (G.19). For the right hand side, substitute for $\tilde{L}, \bar{z}$ and S with equations (C.19), (C.10), and (B.14). Then, use g and $\hat{z}$ from equations (G.21), (G.20), and solve for Ω in terms of model parameters:

$$\Omega = \frac{1}{\zeta}\frac{\chi(1+\theta-\sigma)}{(1-\chi)\theta\sigma\rho}\left(1+(N-1)d^{-\theta}\left(\frac{\kappa\chi}{\zeta\rho(1-\chi)}\right)^{1-\frac{\theta}{\sigma-1}} - \frac{1+\theta-\sigma}{\theta\sigma(1-\chi)}\right)^{-1}. \tag{G.27}$$

Note, from equations (C.47) and (G.20), the home trade share is,

$$\lambda_{ii} = \frac{1}{1+(N-1)d^{-\theta}\left(\frac{\kappa\chi}{\zeta\rho(1-\chi)}\right)^{1-\frac{\theta}{\sigma-1}}}. \tag{G.28}$$

Note, using eqs. G.21 and G.29, the growth rate and Ω can be written as a function of the home trade share:

$$g = \frac{\rho(1-\chi)}{\chi\theta}\frac{\sigma-1}{(\theta-\sigma+1)}\lambda_{ii}^{-1} - \frac{\rho}{\theta}. \tag{G.29}$$

$$\Omega = \frac{\chi}{\zeta\rho}\left(\frac{(1-\chi)\theta\sigma}{1+\theta-\sigma}\lambda_{ii}^{-1} - 1\right)^{-1}. \tag{G.30}$$

Calculating Consumption. By the resource constraint, $c = y$ when $\eta = 0$ (equation C.34). Thus, using equation (C.23), consumption is given by

$$c = y = (1-\tilde{L})\bar{z}. \tag{G.31}$$

Equations (C.19), (B.14), (C.29), and (G.29) combine to yield the amount of labor dedicated to variable goods production in terms of the home trade share:

$$1-\tilde{L} = (\sigma-1)\left(\sigma - \frac{1+\theta-\sigma}{\theta(1-\chi)}\lambda_{ii}\right)^{-1}. \tag{G.32}$$

Equation (C.43) gives

$$\bar{z} = \Omega^{\frac{1}{\sigma-1}}\lambda_{ii}^{\frac{1}{1-\sigma}}\left(\mathbb{E}\left[z^{\sigma-1}\right]\right)^{\frac{1}{\sigma-1}}. \tag{G.33}$$

Substituting equations (G.32), (G.33), and (G.30) into equation (G.31) yields consumption as a function of parameters and the home trade share:

$$c = \frac{(\sigma-1)\theta\sigma(1-\chi)}{\sigma(1+\theta-\sigma)}\left(\frac{\chi}{\rho\zeta}\right)^{\frac{1}{\sigma-1}}\left(\frac{(1-\chi)\theta\sigma}{1+\theta-\sigma} - \lambda_{ii}\right)^{\frac{\sigma}{1-\sigma}}\left(\mathbb{E}\left[z^{\sigma-1}\right]\right)^{\frac{1}{\sigma-1}}. \tag{G.34}$$

Trade Cost Elasticities. Comparative statics are analyzed by calculating elasticities with respect to trade costs using equations (G.21), (G.27), and (G.28):

$$\frac{\partial \log \bar{\pi}_{\text{rat}}(d)}{\partial \log d} = -\theta \left(1 + \frac{\theta d^{\theta} \left(\frac{\kappa\chi}{\zeta\rho(1-\chi)}\right)^{\frac{\theta}{\sigma-1}-1}}{(N-1)(\sigma-1)}\right)^{-1} < 0, \tag{G.35}$$

$$\frac{\partial \log g(d)}{\partial \log d} = -\theta \left(1 + \frac{\rho d^{\theta}(-\theta\chi + \sigma - 1)\left(\frac{\kappa\chi}{\zeta\rho(1-\chi)}\right)^{\frac{\theta}{\sigma-1}}}{\kappa(N-1)(\sigma-1)\chi}\right)^{-1} < 0, \tag{G.36}$$

$$\frac{\partial \log \Omega(d)}{\partial \log d} = \theta \left(1 + \frac{\rho d^{\theta}((\theta+1)(\sigma-1) - \theta\sigma\chi)\left(\frac{\kappa\chi}{\zeta\rho(1-\chi)}\right)^{\frac{\theta}{\sigma-1}}}{\theta\kappa(N-1)\sigma\chi}\right)^{-1} > 0, \tag{G.37}$$

$$\frac{\partial \log \lambda_{ii}(d)}{\partial \log d} = \theta \left(1 + \frac{d^{\theta}}{N-1}\left(\frac{\kappa\chi}{\zeta\rho(1-\chi)}\right)^{\frac{\theta}{\sigma-1}-1}\right)^{-1} > 0. \tag{G.38}$$

Let $\varepsilon_{f,x}$ be the elasticity of any $f(x)$ w.r.t. x. These elasticities can be rearranged to highlight their relationship to trade volume. To see this, first define the ratio of the home trade share to the share of goods purchased away from home:

$$\frac{\lambda_{ii}}{1-\lambda_{ii}} = \frac{d^{\theta}\left(\frac{\kappa\chi}{\zeta\rho(1-\chi)}\right)^{\frac{\theta}{\sigma-1}-1}}{N-1}. \tag{G.39}$$

Substituting this into the result above, yields

$$\frac{\partial \log \lambda_{ii}(d)}{\partial \log d} = \theta\left(1 + \frac{\lambda_{ii}}{1-\lambda_{ii}}\right)^{-1} = \theta(1-\lambda_{ii}), \tag{G.40}$$

$$\frac{\partial \log g(d)}{\partial \log d} = -\theta\left[1 + \left(\frac{-\theta\chi + \sigma - 1}{(\sigma-1)(1-\chi)}\right)\frac{\lambda_{ii}}{1-\lambda_{ii}}\right]^{-1} \tag{G.41}$$

$$= \left(\frac{\chi(1+\theta-\sigma)}{(\sigma-1)(1-\chi)}\lambda_{ii} - 1\right)^{-1} \varepsilon_{\lambda_{ii},d}, \tag{G.42}$$

$$\frac{\partial \log \Omega_{ii}(d)}{\partial \log d} = \left(1 - \frac{1+\theta-\sigma}{\theta\sigma(1-\chi)}\lambda_{ii}\right)^{-1} \varepsilon_{\lambda_{ii},d}. \tag{G.43}$$

The elasticity of $(1-\tilde{L})$ w.r.t. d is

$$\frac{\partial \log(1-\tilde{L}(d))}{\partial \log d} = \left(\frac{\theta\sigma(1-\chi)}{1+\theta-\sigma}\lambda_{ii}^{-1} - 1\right)^{-1} \varepsilon_{\lambda_{ii},d} > 0. \tag{G.44}$$

The elasticity of $\bar{z}$ w.r.t. d is

$$\frac{\partial \log(\bar{z}(d))}{\partial \log d} = \frac{\varepsilon_{\Omega,d} - \varepsilon_{\lambda_{ii},d}}{\sigma - 1} > 0. \tag{G.45}$$

From equation (G.31)

$$\varepsilon_{c,d} = \varepsilon_{1-\tilde{L},d} + \varepsilon_{\bar{z},d}. \tag{G.46}$$

Using equations (G.34) and (G.40) yields

$$\varepsilon_{c,d} = -\frac{\sigma}{\sigma - 1}\left(1 - \frac{\theta\sigma(1-\chi)}{(1+\theta-\sigma)}\lambda_{ii}^{-1}\right)^{-1}\varepsilon_{\lambda_{ii},d}. \tag{G.47}$$

Finally, from D.2

$$\varepsilon_{\bar{U},d} = \frac{\rho\,\varepsilon_{c,d} + g\,\varepsilon_{g,d}}{g + \rho\log(c\bar{L}M(0))} \tag{G.48}$$

$$= \frac{\rho^2}{\bar{U}}\,(\rho\,\varepsilon_{c,d} + g\,\varepsilon_{g,d}). \tag{G.49}$$

This can be further organized by substitution for $\varepsilon_{c,d}$ and $\varepsilon_{g,d}$ from equations (G.47) and (G.42) into equation (G.49).[31]

$$\varepsilon_{\bar{U},d} = -\varepsilon_{\lambda_{ii},d}\frac{\rho^3}{\bar{U}}\left(\frac{\sigma}{(\sigma-1)\left(1 - \frac{\theta\sigma(1-\chi)}{(\theta-\sigma+1)\lambda_{ii}}\right)} + \frac{(\sigma-1)(1-\chi)}{\theta\chi(\theta-\sigma+1)\lambda_{ii}}\right). \tag{G.50}$$

Therefore, if the term in brackets is positive, then the elasticity of utility is of the opposite sign of the home trade share, and hence always decreasing in trade costs.

Firm Adoption Timing. A firm adopts when normalized productivity equals 1 by definition. On the BGP, firms drift backwards towards $z = 1$ at constant rate g. Thus, the time until adoption $\tau(z)$ is given by

$$e^{-g\tau}z = 1$$

$$\tau(z) = \frac{\log(z)}{g} \tag{G.51}$$

[31] For determining the direction of the change, as an elasticity is $d\bar{U}'(d)/\bar{U}(d)$, and since $d > 1$, if $\bar{U}(d) > 0$ then the sign of this elasticity calculation matches the sign of the derivative. Otherwise, the sign of the derivative is negative of the elasticity. As equation (G.49) divided by the utility in the calculation, this sign cancels, and ensures that negative utility does not effect the direction of the changes (as expected with a monotone function with the possibility of arbitrarily small initial conditions).

The expected time until adoption for a firm that is about to draw a new productivity, $\bar{\tau}$, is just the expected adoption time integrated over the distribution of the new z:

$$\bar{\tau} = \int_1^{\infty} \frac{\log(z)}{g} dF(z) = \frac{1}{g}\int_1^{\infty} \log(z)\theta z^{-\theta-1} = \frac{1}{\theta g} \tag{G.52}$$

Since firms draw a new z from the unconditional distribution, the expected time to adoption for a newly adopting firm is the same as the average time to adoption.

H. Computing the BGP Equilibrium in General

In the general case of the $\kappa = 0$, the equilibrium g can be calculated through an explicit equation, and the Ω found separately. If $\kappa > 0$, then a system of 2 non-linear algebraic equations in g and Ω are solved. Summarizing equations for easy reference against the code:

General Substitutions. The following substitutions are used in reducing the equilibrium conditions into a simple system of equations that can be solved for g and Ω. Given g and Ω all other equilibrium values are determined. We use equations (B.13), (B.14), (E.3), (E.4), (E.5), (D.3), (C.19), (C.10), (C.29), (C.12), and (C.36):

$$F(z) = 1 - z^{-\theta} \tag{H.1}$$

$$S = \theta\left(g - \mu - \theta\frac{\upsilon^2}{2}\right) \tag{H.2}$$

$$\nu = \frac{\mu - g}{\upsilon^2} + \sqrt{\left(\frac{g-\mu}{\upsilon^2}\right)^2 + \frac{r-g}{\upsilon^2/2}} \tag{H.3}$$

$$a = \frac{1}{r - g - (\sigma-1)(\mu - g + (\sigma-1)\upsilon^2/2)} \tag{H.4}$$

$$b = (1 - a(r-g))\, d^{1-\sigma}\hat{z}^{\nu+\sigma-1} \tag{H.5}$$

$$r = \rho + \gamma g + \delta \tag{H.6}$$

$$\tilde{L} = \Omega\left[(N-1)(1-F(\hat{z}))\kappa + (1-\eta)\zeta\left(S + \delta/\chi\right)\right] \tag{H.7}$$

$$\bar{z} = \left[\Omega\left(\mathbb{E}\left[z^{\sigma-1}\right] + (N-1)(1-F(\hat{z}))d^{1-\sigma}\mathbb{E}\left[z^{\sigma-1}|z > \hat{z}\right]\right)\right]^{1/(\sigma-1)} \tag{H.8}$$

$$\hat{z} = d\left(\frac{\kappa}{\bar{\pi}_{\min}}\right)^{\frac{1}{\sigma-1}} \tag{H.9}$$

$$w = \frac{1}{\sigma}\bar{z} \tag{H.10}$$

$$x = \zeta\left(1 - \eta + \eta\Theta/w\right) \tag{H.11}$$

(Note, since $\bar{\pi}_{\min}$ is an implicit function through the $\hat{z}$ in $\bar{z}$, it is easiest to add it to the system of equations instead of substituting it out).

All Firms Export Case. For any $\upsilon \geq 0$, the growth rate is given by equation (F.6), substituting for r from D.3.

$$g = \mu + \frac{(r-\mu)((\sigma-1)/\chi - \theta)}{\theta^2 - \theta\sigma + (\sigma-1)/\chi} + \frac{\upsilon^2}{2} \frac{\theta^2(\theta+1-\sigma)^2}{(\theta^2 - \theta\sigma + (\sigma-1)/\chi)(\theta - (\sigma-1)/\chi)}. \tag{H.12}$$

Using the equilibrium g above, Ω can be found by solving the following system of equations in Ω and $\bar{\pi}_{\min}$ from equations (F.3) and (C.26) (where Ω is implicitly in the $\bar{z}$ and $\tilde{L}$ terms):

$$\frac{x}{\bar{\pi}_{\min}} = a\left(1 + (N-1)d^{1-\sigma}\right) \frac{\chi}{1-\chi} \frac{\sigma+\nu-1}{\nu}. \tag{H.13}$$

$$\bar{\pi}_{\min} = \tfrac{1-\tilde{L}}{(\sigma-1)\bar{z}^{\sigma-1}}. \tag{H.14}$$

Selection into Exporting Case. Equations (G.1), (G.3), and (C.26) provide a system of 3 equations in g, Ω, and $\bar{\pi}_{\min}$. To solve this non-linear system, substitute for $\bar{\pi}_{\min}, \nu, a, b, x, r, S, \tilde{L}, \bar{z}$ and $\hat{z}$ using the general substitutions listed above to eliminate dependence on all other endogenous variables.

$$\frac{x}{\bar{\pi}_{\min}} = a \frac{\chi}{1-\chi} \frac{\sigma+\nu-1}{\nu}, \tag{H.15}$$

$$1 + \frac{\sigma-1}{\nu} = \frac{\frac{\nu(n-1)(\theta-\sigma+1)\left(d^{1-\sigma}(\theta+\nu)\hat{z}^{-\theta+\sigma-1} - b\theta\hat{z}^{-\theta-\nu}\right)}{a(g-r)} + \theta\left(\nu(n-1)d^{1-\sigma}(\theta+\nu)\hat{z}^{-\theta+\sigma-1} + (\nu+\sigma-1)(\theta+\nu-\sigma+1)\right)}{\nu(\theta+\nu)(\theta-\sigma+1)} - \frac{\chi}{1-\chi}\frac{\sigma+\nu-1}{\nu}, \tag{H.16}$$

$$\bar{\pi}_{\min} = \tfrac{1-\tilde{L}}{(\sigma-1)\bar{z}^{\sigma-1}}. \tag{H.17}$$

This system of equations holds for both the general case and the baseline case of $\upsilon = \mu = 0$ (if using Mathematica, make sure to substitute ν to reorganize the formulas to avoid any singularity). Alternatively, for the baseline case, together with the same definition of $\bar{\pi}_{\min}$ use equations (G.4) and G.9 as the system of equations, using the same substitutions as in the more general case.

Post Solution Calculations. In either case, given the equilibrium g and Ω, the following equilibrium values can be calculated through equations (C.33), (C.23), (D.1), (D.2), (C.47), and (C.35).

Welfare is normalized to drop dependence on initial conditions and proportional factors.

$$\bar{\pi}_{\text{agg}} = \bar{\pi}_{\min}\bar{z}^{\sigma-1} - \Omega(N-1)(1-F(\hat{z}))\kappa, \tag{H.18}$$

$$y = \left(1-\tilde{L}\right)\bar{z}, \tag{H.19}$$

$$\bar{U} = \begin{cases} \frac{1}{1-\gamma}\frac{c^{1-\gamma}}{\rho+(\gamma-1)g} & \gamma \neq 1 \\ \rho\log(c) + g & \gamma = 1 \end{cases}, \tag{H.20}$$

$$\lambda_{ii} = \frac{1}{1+(N-1)\hat{z}^{\sigma-1-\theta}d^{1-\sigma}}, \tag{H.21}$$

$$c = \left(1-\tilde{L}\right)\bar{z} - \eta\zeta\Omega\Theta\left(S+\delta/\chi\right). \tag{H.22}$$

Consumption Equivalents. Here we focus on the log utility case. Let superscript A represent variables associated with the autarky equilibrium and superscript T denote variables in the trade equilibrium. Time zero utility under autarky is

$$U_0^A = \frac{\rho\log(c_o^A) + g^A}{\rho^2}. \tag{H.23}$$

We want to know how much extra consumption is required in the autarky equilibrium to make the agent indifferent between living in the autarky and trade equilibrium. That is, we want to find μ such that

$$U_0^A = \frac{\rho\log(\mu c_o^A) + g^A}{\rho^2} = \frac{\rho^2\log(c_o^T) + g^T}{\rho^2}. \tag{H.24}$$

The μ that solves this relationship takes a simple form:

$$\mu = \exp\left(\frac{U_0^T - U_0^A}{\rho}\right). \tag{H.25}$$

I. Notation

General notation principle for normalization: move to lowercase after normalizing to the scale of the economy, from nominal to real, per-capita, and relative wages (all where appropriate). For symmetric countries, denote variables related to the trade sector with an x subscript. An overbar denotes an aggregation of the underlying variable. Drop the t subscript where possible for clarity in the static equilibrium conditions.

Notation Summary

Parameters	
Multiplicative Adoption Cost	$\zeta > 0$
Labor/Goods Proportion of Adoption Cost	$0 \leq \eta \leq 1$
Goods Adoption Cost	$\Theta \geq 0$
Iceberg Trade Cost	$d \geq 1$
Fixed Export Cost	$\kappa \geq 0$
Consumer CRRA	$\gamma \geq 0$
Elasticity of Substitution between Products	$\sigma > 1$
Markup	$\bar{\sigma} := \sigma/(\sigma - 1)$
Tail Parameter of the Pareto Distribution	$\theta > 1$
Consumer's Discount Factor	$\rho > 0$
Population per Country	$\bar{L} > 0$
Adoption to Entry Cost Ratio	$\chi < 1$
Firm Death Rate	$\delta \geq 0$
Drift of Productivity Process	$\mu \geq 0$
Standard Deviation of Productivity Process	$\upsilon \geq 0$

With the simplest baseline model, and ignoring parameters which only effect the economy scale, the minimal number of parameters to calibrate is: $d, \kappa, \sigma, \rho, \theta$, and χ.

Notation Summary

Equilibrium Variables	
Productivity	Z
CDF of the Productivity Distribution	$\Phi(Z,t)$
PDF of the Productivity Distribution	$\phi(Z,t)$
Representative Consumers Flow Utility	$U(t)$
Representative Consumers Welfare	$\bar{U}(t)$
Real Firm Value	$V(Z,t)$
Optimal Search Threshold	$M(t)$
Optimal Export Threshold	$\hat{Z}(t)$
Aggregate Nominal Expenditures on Final Goods	$Y(t)$
Aggregate Real Consumption	$C(t)$
Domestic Labor demand	$\ell_d(Z,t)$
Export Labor demand	$\ell_x(Z,t)$
Domestic Quantity	$Q_d(Z,t)$
Export Quantity	$Q_x(Z,t)$
Real Search Cost	$X(t)$
Domestic idiosyncratic prices	$p_d(Z,t)$
Export idiosyncratic prices	$p_x(Z,t)$
Nominal Wages	$W(t)$
Real Domestic Profits	$\Pi_d(Z,t)$
Real Per-market Export Profits	$\Pi_x(Z,t)$
Interest Rate	$r(t)$
Trade Share	$\lambda(t)$
Price level	$P(t)$
Number of Varieties	$\Omega(t)$

Normalization Notation Summary (implicit t where appropriate)

Real, Normalized, and Per-Capita Variables	
Per-capita Labor Demand/Supply	$L := \bar{L}/\bar{L}$
Normalized Productivity	$z := Z/M$
Normalized Optimal Export Threshold	$\hat{z} := \hat{Z}/M$
Normalized CDF of the Productivity Distribution	$F(z,t) := \Phi(zM(t),t)$
Normalized PDF of the Productivity Distribution	$f(z,t) := M(t)\phi(zM(t),t)$
Expectation of the Normalized Productivity Distribution	$\mathbb{E}\left[\Psi(z)\right] := \int_1^\infty \Psi(z)f(z)\mathrm{d}z$
Conditional Expectation of the Normalized Productivity Distribution	$\mathbb{E}\left[\Psi(z)\vert z > \hat{z}\right] := \int_{\hat{z}}^\infty \Psi(z)\frac{f(z)}{1-F(\hat{z})}\mathrm{d}z$
Normalized, Per-capita Real Firm Value Normalized by Real Wages	$v(z,t) := \frac{1}{LMw}V(Z,t)$
Normalized, Per-capita, Real Expenditures on Final Goods (i.e., Output)	$y := \frac{1}{LMP}Y$
Normalized, Per-capita Real consumption	$c := \frac{1}{LM}C$
Normalized, Per-capita, Real Adoption Cost Relative to Real Wages	$x := \frac{1}{LMw}X$
Normalized Real Wages	$w := \frac{1}{PM}W$
Normalized, Per-capita, Real, Aggregate Domestic Profits	$\bar{\pi}_d$
Normalized, Per-capita, Real, Aggregate Per-market Export Profits	$\bar{\pi}_x$